

# Package ‘genpca’

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**Type** Package

**Title** Generalized Principal Component Analysis

**Version** 0.2.2

**Description** Generalized PCA and related matrix decompositions in weighted inner-product spaces. Methods are based on Allen, G. I., Grose, L., and Taylor, J. (2014) <[doi:10.1080/01621459.2013.852978](https://doi.org/10.1080/01621459.2013.852978)>, ``A generalized least-square matrix decomposition'', Journal of the American Statistical Association, 109(505), 145-159; and Abdi, H. (2007), ``Singular value decomposition (SVD) and generalized singular value decomposition" <<https://personal.utdallas.edu/~herve/Abdi-SVD2007-pretty.pdf>>, in ``Encyclopedia of Measurement and Statistics'', 907-912.

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|            |                                                        |
|------------|--------------------------------------------------------|
| geigen_cov | <i>Generalized eigenproblem on a covariance matrix</i> |
|------------|--------------------------------------------------------|

---

Description

Maximises  $v' C v$  subject to  $v' R v = 1$  and the additional constraint that  $v$  lies in the retained range of  $R$ . Successive components are  $R$ -orthogonal. If  $P$  is the orthogonal projector onto that range, the returned vectors satisfy  $P C v = \lambda R v$  and  $V' R V = I$ . For a full-rank  $R$  this is the usual equation  $C v = \lambda R v$ . It also holds for a singular  $R$  when  $C$  maps its retained range into itself. Otherwise the component of  $C v$  outside the retained range need not vanish.

Usage

```
geigen_cov(  
  C,  
  R = NULL,  
  ncomp = NULL,  
  constraints_remedy = c("error", "ridge", "clip", "identity"),  
  rank_rtol = 1e-06,
```

```
metric_rtol = .metric_rtol_default(),
verbose = FALSE
)
```

## Arguments

|                    |                                                                                                                                                                                                                                                                                                                                              |
|--------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| C                  | A $p \times p$ symmetric matrix. Asymmetry beyond roundoff is an error; an indefinite C is allowed (the problem is still defined) and only produces a warning.                                                                                                                                                                               |
| R                  | Variable-side constraint/metric. Can be: <ul style="list-style-type: none"> <li>• NULL: identity matrix (standard PCA on C)</li> <li>• a numeric vector of length <math>p</math>: diagonal weights (must be non-negative)</li> <li>• a <math>p \times p</math> symmetric PSD matrix: general metric/smoothing/structure penalties</li> </ul> |
| ncomp              | Number of components to return. Default is all positive eigenvalues.                                                                                                                                                                                                                                                                         |
| constraints_remedy | What to do with an indefinite R: "error" (default), "ridge", "clip" or "identity"; a repair emits a <code>genpca_metric_repaired</code> warning. See <a href="#">genpca</a> .                                                                                                                                                                |
| rank_rtol          | Relative cutoff for component acceptance on the singular-value scale (components with $d_j \leq \text{rank\_rtol} * d_1$ are dropped). Default $1e-6$ .                                                                                                                                                                                      |
| metric_rtol        | Relative tolerance for validating C and R and for detecting the numerical null space in an eigendecomposition of a general R. Every strictly positive diagonal weight is retained without a rank approximation. Default $\sqrt{\text{.Machine\$double.eps}}$ .                                                                               |
| verbose            | Logical. If TRUE, print progress messages. Default FALSE.                                                                                                                                                                                                                                                                                    |

## Details

This is a different estimator from the GMD of [genpca\\_cov](#), which uses  $R^{1/2}CR^{1/2}$ . If C and R commute, their common eigenvectors can be ordered differently: the GMD weights variances by metric eigenvalues, whereas this estimator divides by them. With  $R = c * I$ , the directions and their ordering agree, but the eigenvalue scales differ unless  $c = 1$ .

C is validated for symmetry but may be indefinite (the generalized eigenproblem is still defined); a warning is issued when its minimum eigenvalue is below  $-\text{metric\_rtol} * \text{scale}$ . R must be positive semi-definite; an indefinite R is subject to `constraints_remedy`.

## Value

A plain list with the same components as [genpca\\_cov](#) (`v`, `d`, `lambda`, `k`, `propv`, `cumv`, `R_rank`) and `method = "geigen"`. `propv` is relative to  $\text{tr}(R^{-1/2}CR^{-1/2})$  on the range of R.

## See Also

[genpca\\_cov](#)

## Examples

```
C <- cov(scale(iris[,1:4], center=TRUE, scale=FALSE))
w <- c(1, 1, 0.5, 2)
fit_gmd <- genpca_cov(C, R = w, ncomp = 2)
fit_geigen <- geigen_cov(C, R = w, ncomp = 2)
# different estimators: the singular values generally differ
rbind(gmd = fit_gmd$d, geigen = fit_geigen$d)

# With singular R, the equation is projected onto its retained range
C <- matrix(c(2, 1, 1, 2), 2)
R <- diag(c(1, 0))
fit <- geigen_cov(C, R, ncomp = 1)
P <- diag(c(1, 0))
P %%% C %%% fit$v - (R %%% fit$v) * fit$lambda
```

---

genpca

*Generalised Principal Components Analysis (GPCA)*


---

## Description

Implements the Generalised Least-Squares Matrix Decomposition of Allen, Groseknick & Taylor (2014) for data observed in a **row** inner-product space **M** and a **column** inner-product space **A**. Setting  $M = I_n$ ,  $A = I_p$  recovers ordinary PCA.

## Usage

```
genpca(
  X,
  A = NULL,
  M = NULL,
  ncomp = NULL,
  method = c("eigen", "auto", "spectra", "randomized", "deflation"),
  constraints_remedy = c("error", "ridge", "clip", "identity"),
  preproc = multivarious::pass(),
  threshold = 1e-06,
  maxit_deflation = 500L,
  use_cpp = TRUE,
  maxeig = 5000,
  warn_approx = TRUE,
  maxit_spectra = 1000,
  tol_spectra = 1e-09,
  rank_rtol = 1e-06,
  oversample = 20L,
  n_power = 1L,
  n_polish = 0L,
  jitter_metric = 1e-10,
  seed_randomized = 1234L,
```

```

    tol_polish_randomized = 1e-04,
    verbose = FALSE
  )

```

## Arguments

|                                 |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |
|---------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>X</code>                  | Numeric matrix $n \times p$ .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |
| <code>A</code>                  | Column constraint: vector (implies diagonal), dense matrix, or sparse symmetric $p \times p$ PSD matrix. If NULL, defaults to identity.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |
| <code>M</code>                  | Row constraint: vector (implies diagonal), dense matrix, or sparse symmetric $n \times n$ PSD matrix. If NULL, defaults to identity.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |
| <code>ncomp</code>              | Number of components to extract. Defaults to $\min(\dim(X))$ . Must be positive.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |
| <code>method</code>             | Character string specifying the computation method. One of "eigen" (default, uses gmdLA), "auto" (heuristic choice among "eigen", "spectra", and "randomized"), "spectra" (iterative partial SVD of the metric-whitened data via <b>eigencore</b> , gmd_spectra), "randomized" (approximate randomized block solver gmd_randomized), or "deflation" (uses gmd_deflationR or gmd_deflation_cpp).                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |
| <code>constraints_remedy</code> | Character string specifying what to do with a supplied A or M that is not positive semi-definite (within a relative tolerance of $\sqrt{\text{.Machine$double.eps}}$ ). Default "error": reject the input. The alternatives repair it and emit a warning of class <code>genpca_metric_repaired</code> whose report field (see <a href="#">repair_metric</a> ) records the minimum eigenvalue before and after, the shift applied, the rank and the condition number: "ridge" (Gershgorin diagonal shift: add the smallest diagonal loading that restores positive definiteness, falling back to <code>Matrix::nearPD()</code> for small dense matrices), "clip" (spectral clip to the PSD cone by zeroing negative eigenvalues; this densifies the matrix and refuses sparse input larger than 2000 rows/cols, where "ridge" should be used instead), or "identity" (replace the matrix with the identity). An asymmetric metric is an error under every setting. Singular PSD metrics are valid input and are never repaired. |
| <code>preproc</code>            | Pre-processing transformer object from the <b>multivarious</b> package (default <code>multivarious::pass()</code> ). Use <code>multivarious::center()</code> for centered GPCA. See <code>?multivarious::prep</code> for options.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |
| <code>threshold</code>          | Convergence tolerance for the "deflation" method's inner loop. Default $1e-6$ . Cutoffs are relative to the scale of the problem (the norm/singular-value floors scale with $\sqrt{\text{tr}(X'MXA)}$ ), so results are invariant to rescaling $X$ . The convergence check is on a squared step difference, so the resulting singular-vector accuracy scales like $\sqrt{\text{threshold}}$ , not threshold itself.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |
| <code>maxit_deflation</code>    | Maximum iterations per component for the "deflation" method. Default 500.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |
| <code>use_cpp</code>            | Logical. If TRUE (default) and package was compiled with C++ support, use faster C++ implementation for <code>method = "deflation"</code> . Fallback to R otherwise. (Ignored for <code>method = "eigen"</code> and <code>method = "spectra"</code> ).                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |
| <code>maxeig</code>             | For <code>method = "eigen"</code> and <code>method = "spectra"</code> : a positive definite general metric is factored exactly by Cholesky at any size, but a singular general metric (e.g. a graph Laplacian) needs a dense eigendecomposition of the metric,                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |

|                                    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |
|------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|                                    | which is refused when the metric has more than <code>maxeig</code> rows. The error names the alternatives ( <code>method = "deflation"</code> , which only multiplies by the metric, or raising <code>maxeig</code> ); <code>method = "auto"</code> routes such cases to deflation. Results are never approximated. Default 5000.                                                                                                                                                                                                                                                                     |
| <code>warn_approx</code>           | Deprecated and ignored: <code>method = "eigen"</code> no longer approximates anything.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |
| <code>maxit_spectra</code>         | Retained for compatibility and currently unused: the <b>eigencore</b> partial SVD used by <code>method = "spectra"</code> is controlled by <code>tol_spectra</code> alone.                                                                                                                                                                                                                                                                                                                                                                                                                            |
| <code>tol_spectra</code>           | Convergence tolerance of the iterative solver when <code>method = "spectra"</code> . Default $1e-9$ . This governs iteration only; rank decisions use <code>rank_rtol</code> .                                                                                                                                                                                                                                                                                                                                                                                                                        |
| <code>rank_rtol</code>             | Relative cutoff for component acceptance, on the scale of the singular values: component $j$ is dropped when $d_j \leq \text{rank\_rtol} * d_1$ . Applied by every method (for the eigen paths on $d_j^2$ ), so the number of components returned does not change when $X$ is rescaled. Default $1e-6$ . Metric validation uses a separate relative tolerance, <code>sqrt(.Machine\$double.eps)</code> , for positive semi-definiteness and null-space detection for general metric eigendecompositions. Every strictly positive diagonal weight is retained in both the forward and inverse factors. |
| <code>oversample</code>            | Oversampling for <code>method = "randomized"</code> ( <code>sketch size = ncomp + oversample</code> ). Default 20.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |
| <code>n_power</code>               | Number of power iterations for <code>method = "randomized"</code> . Default 1.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |
| <code>n_polish</code>              | Number of optional block-polish iterations for <code>method = "randomized"</code> . Default 0.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |
| <code>jitter_metric</code>         | Relative Gram jitter for the candidate Cholesky preconditioner in <code>method = "randomized"</code> . The basis is checked in the original metric; a failed check uses rank-revealing orthonormalization instead. Default $1e-10$ .                                                                                                                                                                                                                                                                                                                                                                  |
| <code>seed_randomized</code>       | Optional seed for <code>method = "randomized"</code> . Default 1234. This fully determines the randomized backend's random stream: the C++ kernel seeds its own generator from this value rather than from <code>R's set.seed()/Random.seed</code> , and calling <code>genpca()</code> with <code>method = "randomized"</code> does not alter the caller's <code>.Random.seed</code> . To reproduce a randomized fit, fix <code>seed_randomized</code> , not the R seed.                                                                                                                              |
| <code>tol_polish_randomized</code> | Relative tolerance used for early stopping of polish iterations in <code>method = "randomized"</code> . Set 0 to disable early stop. Default $1e-4$ .                                                                                                                                                                                                                                                                                                                                                                                                                                                 |
| <code>verbose</code>               | Logical. If TRUE, print progress messages. Default FALSE.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |

### Value

An object of class `c("genpca", "bi_projector")` inheriting from `multivarious::bi_projector`, with slots including:

**u,v** Left/right singular vectors scaled by the constraint metrics (MU, AV). These correspond to components in the original space's geometry. Use `components(fit)`.

**ou,ov** Orthonormal singular vectors in the constraint metric (U, V such that  $U^T M U = I$ ,  $V^T A V = I$ ). These are the core mathematical factors.

- sdev** Generalised singular values  $d_k$ . Note these are singular values of the metric-whitened data matrix, not standard deviations: with identity metrics and centering,  $sdev = prcomp(X)$sdev * sqrt(nrow(X) - 1)$ .
- s** Scores: the generalised principal components  $z_k = X A$   $ov_k = ou_k d_k$  (Allen et al. 2014, Section 2.4). Identical to `project(fit, X)` on the training data. Use `scores(fit)`.
- preproc** The multivarious pre-processing object used.
- A, M** The constraint matrices used (potentially after coercion to sparse format).
- propv** Proportion of generalized variance explained by each component.
- cumv** Cumulative proportion of generalized variance explained.

## Method

We compute the rank-ncomp factors  $UDV^T$  that minimise

$$\|X - UDV^T\|_{M,A}^2 = \text{tr}(M(X - UDV^T)A(X - UDV^T)^T)$$

subject to  $U^T M U = I$ ,  $V^T A V = I$ . (Allen et al., 2014). Five methods are available via the `method` argument:

- "eigen" (Default): Uses a one-shot eigen decomposition strategy based on `gmdLA`. It explicitly forms and decomposes a  $p \times p$  or  $n \times n$  matrix (depending on  $n$  vs  $p$ ).
- "auto": Chooses among "eigen", "spectra", and "randomized" using heuristics on shape, rank ratio ( $ncomp / \min(n,p)$ ), and constraint structure.
- "spectra": Computes the top-k singular triplets of the metric-whitened data  $F'_M X F_A$  (with  $M = F_M F'_M$ ,  $A = F_A F'_A$ ) as an implicit operator via the **eigencore** package, without forming the large intermediate matrix. Generally faster and uses less memory for large  $n$  or  $p$  when few components are requested.
- "randomized": Uses a randomized block range finder and small projected eigendecomposition. This is an approximate low-pass method that is often much faster for wide dense matrices with sparse metrics when only top components are needed.
- "deflation": Uses an iterative power/deflation algorithm. Can be slower but potentially uses less memory than "eigen" for very large dense problems where  $ncomp$  is small.

## Backend Guidance

The default is `method = "eigen"`; "auto" is opt-in, not the default.

- Use "eigen" (the default) when you need a stable reference solution on small/medium problems.
- Use "auto" to let a heuristic pick among "eigen", "spectra", and "randomized" based on problem shape and constraint structure.
- Use "spectra" for larger matrix-free iterative solves where memory pressure is a concern.
- Use "randomized" for wide low-rank settings ( $p \gg n$ ) with sparse metrics when throughput matters most.
- Use "deflation" when you only need a few components and can tolerate iterative convergence behavior.

For pre-computed covariance matrices  $C = X'MX$ , see [genpca\\_cov](#) which performs GPCA directly on  $C$  with column constraint  $R$  (equivalent to  $A$ ).

## References

Allen, G. I., Grose, L., & Taylor, J. (2014). *A Generalized Least-Squares Matrix Decomposition*. Journal of the American Statistical Association, 109(505), 145-159. arXiv:1102.3074.

## See Also

[genpca\\_cov](#) for GPCA on pre-computed covariance matrices, [truncate.genpca](#), [reconstruct.genpca](#), [multivarious::bi\\_projector](#), [multivarious::project](#), [multivarious::scores](#), [multivarious::components](#), [multivarious::reconstruct](#).

## Examples

```
if (requireNamespace("multivarious", quietly = TRUE)) {
  set.seed(123)
  X <- matrix(stats::rnorm(200 * 100), 200, 100)
  rownames(X) <- paste0("R", 1:200)
  colnames(X) <- paste0("C", 1:100)

  # Standard PCA (A=I, M=I, centered) - using default method="eigen"
  gpca_std_eigen <- genpca(X, ncomp = 5, preproc = multivarious::center(), verbose = FALSE)

  # Standard PCA using Spectra method (requires C++ build)
  # gpca_std_spectra <- try(genpca(X, ncomp = 5,
  #                             preproc = multivarious::center(),
  #                             method = "spectra", verbose = TRUE))
  # if (!inherits(gpca_std_spectra, "try-error")) {
  #   print(head(gpca_std_spectra$sdev))
  # }

  # Compare singular values with prcomp
  pr_std <- stats::prcomp(X, center = TRUE, scale. = FALSE)
  print("Eigen Method Sdev:")
  print(head(gpca_std_eigen$sdev))
  print("prcomp Sdev:")
  print(head(pr_std$sdev))
  print(paste("Total Var Explained (Eigen):",
              round(sum(gpca_std_eigen$propv) * 100, "%")))

  # Weighted column PCA (diagonal A, no centering)
  col_weights <- stats::runif(100, 0.5, 1.5)
  gpca_weighted <- genpca(X, A = col_weights, ncomp = 3,
                          preproc = multivarious::pass(), verbose = FALSE)
  print("Weighted GPCA Sdev:")
  print(gpca_weighted$sdev)
  print(head(components(gpca_weighted)))
}
```

genpca\_cov

*Generalized PCA on a covariance matrix (GMD form)***Description**

Performs Generalized PCA directly on a pre-computed covariance matrix  $C = X'MX$  with a single variable-side metric  $R$ , following Allen et al.'s GMD: the eigendecomposition of  $R^{1/2}CR^{1/2}$  mapped back with  $V = R^{-1/2}Z$ , so that  $V'RV = I$ . With  $C = X'MX$  and  $R = A$  this matches [genpca](#)( $X, M = M, A = A$ ) exactly. This is useful when you already have  $C$  or when  $X$  is too large to store but  $C$  is manageable.

**Usage**

```
genpca_cov(
  C,
  R = NULL,
  ncomp = NULL,
  method = c("gmd", "geigen"),
  constraints_remedy = c("error", "ridge", "clip", "identity"),
  rank_rtol = 1e-06,
  metric_rtol = .metric_rtol_default(),
  tol = NULL,
  verbose = FALSE
)
```

**Arguments**

|                    |                                                                                                                                                                                                                                                                                                                                                                        |
|--------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| C                  | A $p \times p$ symmetric positive semi-definite covariance matrix, typically $C = X'MX$ . Asymmetry beyond roundoff and indefiniteness beyond <code>metric_rtol</code> are errors.                                                                                                                                                                                     |
| R                  | Variable-side constraint/metric. Can be: <ul style="list-style-type: none"> <li>• <code>NULL</code>: identity matrix (standard PCA on <math>C</math>)</li> <li>• a numeric vector of length <math>p</math>: diagonal weights (must be non-negative)</li> <li>• a <math>p \times p</math> symmetric PSD matrix: general metric/smoothing/structure penalties</li> </ul> |
| ncomp              | Number of components to return. Default is all positive eigenvalues.                                                                                                                                                                                                                                                                                                   |
| method             | Deprecated. "gmd" (default) is this function; "geigen" forwards to <a href="#">geigen_cov</a> with a warning.                                                                                                                                                                                                                                                          |
| constraints_remedy | Deprecated here (GMD requires PSD input and stops otherwise); forwarded to <a href="#">geigen_cov</a> when <code>method = "geigen"</code> .                                                                                                                                                                                                                            |
| rank_rtol          | Relative cutoff for component acceptance on the singular-value scale (components with $d_j \leq \text{rank\_rtol} * d_1$ are dropped). Default <code>1e-6</code> .                                                                                                                                                                                                     |

|                          |                                                                                                                                                                                                                                                                   |
|--------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>metric_rtol</code> | Relative tolerance for validating C and R and for detecting the numerical null space in an eigendecomposition of a general R. Every strictly positive diagonal weight is retained without a rank approximation. Default <code>sqrt(.Machine\$double.eps)</code> . |
| <code>tol</code>         | Deprecated; use <code>rank_rtol</code> and <code>metric_rtol</code> .                                                                                                                                                                                             |
| <code>verbose</code>     | Logical. If TRUE, print progress messages. Default FALSE.                                                                                                                                                                                                         |

## Details

The generalized eigenproblem  $Cv = \lambda Rv$  is a different estimator (it maximises  $v'Cv$  subject to  $v'Rv = 1$ , which is generally gives different components from the GMD) and lives in its own function, [geigen\\_cov](#). `method = "geigen"` is accepted here for one release and forwards to it with a deprecation warning.

## Value

A plain list (**not** a **multivarious** `bi_projector`) with components:

**v**  $p \times k$  matrix of loadings (R-orthonormal eigenvectors)

**d** Singular values (square root of eigenvalues `lambda`)

**lambda** Eigenvalues (variances under the R-metric)

**k** Number of components returned

**propv** Proportion of variance explained by each component (total variance is  $\text{tr}(CR)$ , Allen et al. Corollary 5)

**cumv** Cumulative proportion of variance explained

**R\_rank** Rank of the constraint matrix R

**method** "gmd"

Because this is a plain list rather than a `bi_projector`, the multivarious generics `scores()`, `components()`, and `reconstruct()` do not apply to it; index `$v/$d` directly, or use [genpca](#) when you need the full projector interface on a data matrix rather than a pre-computed covariance matrix.

## References

Allen, G. I., Grotenick, L., & Taylor, J. (2014). A Generalized Least-Squares Matrix Decomposition. *Journal of the American Statistical Association*, 109(505), 145-159.

## See Also

[geigen\\_cov](#) for the generalized eigenproblem  $Cv = \lambda Rv$ , [genpca](#) for the two-sided GPCA on data matrices, [genpls](#) for generalized partial least squares

## Examples

```
# Standard PCA on a covariance (no constraint)
C <- cov(scale(iris[,1:4], center=TRUE, scale=FALSE))
fit0 <- genpca_cov(C, R=NULL, ncomp=3)
print(fit0$d[1:3])      # first 3 singular values
print(fit0$propv[1:3])  # variance explained by first 3 components
```

```

# Equivalence with genpca()
set.seed(123)
X <- matrix(rnorm(50 * 10), 50, 10)
M_diag <- runif(50, 0.5, 1.5) # row weights
A_diag <- runif(10, 0.5, 2)    # column weights
fit_gpca <- genpca(X, M = M_diag, A = A_diag, ncomp = 5,
                  preproc = multivarious::pass())
C <- crossprod(X, diag(M_diag) %*% X) # C = X'MX
fit_cov <- genpca_cov(C, R = A_diag, ncomp = 5)
all.equal(fit_gpca$sdev, fit_cov$d, tolerance = 1e-10)

# Variable weights via a diagonal metric (iris covariance, 4 variables)
C_iris <- cov(scale(iris[,1:4], center=TRUE, scale=FALSE))
w <- c(1, 1, 0.5, 2)
fitW <- genpca_cov(C_iris, R = w, ncomp=3)
print(fitW$d[1:3])

```

---

genpls

---

Generalized PLS via Implicit Operator (PLS-SVD / GPLSSVD)

---

## Description

Canonical (two-block) generalized PLS using sparse-friendly implicit matrix-vector products. Solves the SVD of the operator  $S = Xe'Ye$  without materializing  $Xe = Mx^{1/2}XA x^{1/2}$  or  $Ye = My^{1/2}YA y^{1/2}$ .

## Usage

```

genpls(
  X,
  Y,
  Ax = NULL,
  Ay = NULL,
  Mx = NULL,
  My = NULL,
  ncomp = 2,
  preproc_x = multivarious::pass(),
  preproc_y = multivarious::pass(),
  svd_backend = c("eigencore", "irlba", "RSpectra"),
  svd_opts = list(tol = 1e-07, maxitr = 1000),
  constraints_remedy = c("error", "ridge", "clip", "identity"),
  verbose = FALSE
)

```

## Arguments

|                      |                                                                                                                                                                                                                                                                                                                                                              |
|----------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| X                    | Numeric or Matrix, $n \times p$ .                                                                                                                                                                                                                                                                                                                            |
| Y                    | Numeric or Matrix, $n \times q$ . Must have same $n$ as $X$ .                                                                                                                                                                                                                                                                                                |
| Ax                   | Column metric for $X$ ( $W_X$ ): vector/diagonal/matrix; NULL means identity.                                                                                                                                                                                                                                                                                |
| Ay                   | Column metric for $Y$ ( $W_Y$ ): vector/diagonal/matrix; NULL means identity.                                                                                                                                                                                                                                                                                |
| Mx                   | Row metric for $X$ ( $M_X$ ): vector/diagonal/matrix; NULL means identity.                                                                                                                                                                                                                                                                                   |
| My                   | Row metric for $Y$ ( $M_Y$ ): vector/diagonal/matrix; NULL means identity.                                                                                                                                                                                                                                                                                   |
| ncomp                | Number of components to extract (rank- $k$ ). Default 2.                                                                                                                                                                                                                                                                                                     |
| preproc_x, preproc_y | Optional multivarious preprocessors (e.g., <code>center()</code> ). Defaults to <code>multivarious::pass()</code> (no-op).                                                                                                                                                                                                                                   |
| svd_backend          | Character, one of "eigencore" (default) or "irlba" for the iterative SVD. This choice only matters for larger problems: whenever both $X$ and $Y$ have at most 64 columns after preprocessing, the operator materializes $S$ densely and computes a direct <code>svd()</code> , ignoring <code>svd_backend</code> entirely (see <code>gplssvd_op()</code> ). |
| svd_opts             | List of options: <code>tol</code> for both backends and <code>maxitr</code> for irlba only. An incomplete eigencore solve raises an error of class <code>genpca_solver_nonconvergence</code> ; no unchecked fit is returned.                                                                                                                                 |
| constraints_remedy   | What to do with a metric that is not positive semi-definite: "error" (default), "ridge", "clip" or "identity"; repairs emit a <code>genpca_metric_repaired</code> warning. See <code>genpca()</code> .                                                                                                                                                       |
| verbose              | Logical; print brief progress messages.                                                                                                                                                                                                                                                                                                                      |

## Details

This follows the GPLSSVD/PLS-SVD formulation (Beaton, eqs. 10–14): the top `ncomp` singular triplets of  $S$  are computed by iterative SVD on the linear maps  $v \rightarrow S v$  and  $u \rightarrow S^T u$ , implemented with metric Cholesky multiplies/solves when possible. Works with dense or sparse Matrix inputs and constraint metrics.

Returns a `multivarious::cross_projector` with  $X$ -/ $Y$ -weights ( $v_x, v_y$ ) chosen to provide natural projection of new data ( $X \%*\% v_x, Y \%*\% v_y$ ). Additional GPLSSVD quantities are attached to the object for access: singular values  $d$ , generalized weights  $p, q$ , variable scores  $f_i, f_j$ , and row latent variables  $l_x, l_y$ .

`genpls()` maximizes the covariance between latent variables of  $X$  and  $Y$  under the ( $M_x, A_x, M_y, A_y$ ) metrics by computing the SVD of  $S = X e' Y e$ , where  $X e = M_x^{1/2} X A_x^{1/2}$  and  $Y e = M_y^{1/2} Y A_y^{1/2}$ .

`project()` on a fitted object returns latent variables in the ambient (original data) metric, i.e.  $X \%*\% v_x = X W_X p$ . This differs from the attached  $l_x = M_x^{1/2} X W_X p$ , which lives in the row-whitened metric, by the factor  $M_x^{1/2}$ :  $l_x$  and `project(fit, X)` are equal only when  $M_x = I$ . New-row projection necessarily uses `project()`'s ambient-metric convention, because a training-row metric  $M_x$  has no natural extension to out-of-sample rows.

**Metric naming.** `genpls()`'s row/column metric arguments ( $M_x, M_y$  for rows;  $A_x, A_y$  for columns) follow the same M/A convention as `genpca()`. Internally they are forwarded to `gplssvd_op()`, which uses the names XLW/YLW (left/row weights, i.e.  $M_x/M_y$ ) and XRW/YRW (right/column weights, i.e.  $A_x/A_y$ ).

**Value**

An object of class `c("genpls", "cross_projector", "projector")` with:

**vx, vy** X- and Y- projection weights (stored in `cross_projector`) such that `project(fit, X) = X \%*\% vx` and `project(fit, Y, source = "Y") = Y \%*\% vy` recover the latent variables in the ambient (non-whitened) metric. Algebraically  $vx = W_X p = f_i \%*\% \text{diag}(1/d)$  and  $vy = W_Y q = f_j \%*\% \text{diag}(1/d)$  (see Details).

**d** singular values of  $S = X e' Y e$  (attached field)

**p, q** generalized weights  $W_X^{-1/2} u, W_Y^{-1/2} v$  (attached)

**fi, fj** variable/component scores  $W_X p D, W_Y q D$  (attached)

**lx, ly** row latent variables  $M_X^{1/2} X W_X p, M_Y^{1/2} Y W_Y q$  (attached)

**metrics** the supplied metrics (attached)

**ncomp** Number of components actually extracted. The underlying operator may return fewer than the requested `ncomp` (e.g. when `ncomp` exceeds  $\min(\text{ncol}(X), \text{ncol}(Y))$ ); this field reflects the actual count, not the request.

**backend** The `svd_backend` value passed in (for reference only; see the `svd_backend` argument for when it is actually used).

**preproc\_x, preproc\_y** The fitted multivarious preprocessing objects for X and Y, stored on the `cross_projector`.

**References**

Beaton, D. (2020). Generalized eigen, singular value, and partial least squares decompositions: The GSVD package. (Eqs. 10-14). [arXiv:2010.14734](https://arxiv.org/abs/2010.14734).

**Examples**

```
if (requireNamespace("multivarious", quietly = TRUE)) {
  set.seed(1)
  n <- 100; p <- 40; q <- 30
  X <- matrix(rnorm(n*p), n, p)
  Y <- matrix(rnorm(n*q), n, q)
  w <- runif(n); w <- w/sum(w)
  Mx <- My <- Matrix::Diagonal(x = w)
  fit <- genpls(X, Y, Mx = Mx, My = My, ncomp = 2,
               preproc_x = multivarious::center(),
               preproc_y = multivarious::center())
  fit$d # singular values
}
```

---

genplsc

*Canonical Generalized PLS (alias)*


---

## Description

Convenience alias for `genpls()`; computes canonical generalized PLS (PLS-SVD/GPLSSVD). See `?genpls` for full documentation.

## Usage

```
genplsc(
  X,
  Y,
  Ax = NULL,
  Ay = NULL,
  Mx = NULL,
  My = NULL,
  ncomp = 2,
  preproc_x = multivarious::pass(),
  preproc_y = multivarious::pass(),
  svd_backend = c("eigencore", "irlba", "RSpectra"),
  svd_opts = list(tol = 1e-07, maxitr = 1000),
  constraints_remedy = c("error", "ridge", "clip", "identity"),
  verbose = FALSE
)
```

## Arguments

|                      |                                                                                                                                                                                                                                                                                                                                                        |
|----------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| X                    | Numeric or Matrix, $n \times p$ .                                                                                                                                                                                                                                                                                                                      |
| Y                    | Numeric or Matrix, $n \times q$ . Must have same $n$ as X.                                                                                                                                                                                                                                                                                             |
| Ax                   | Column metric for X ( $W_X$ ): vector/diagonal/matrix; NULL means identity.                                                                                                                                                                                                                                                                            |
| Ay                   | Column metric for Y ( $W_Y$ ): vector/diagonal/matrix; NULL means identity.                                                                                                                                                                                                                                                                            |
| Mx                   | Row metric for X ( $M_X$ ): vector/diagonal/matrix; NULL means identity.                                                                                                                                                                                                                                                                               |
| My                   | Row metric for Y ( $M_Y$ ): vector/diagonal/matrix; NULL means identity.                                                                                                                                                                                                                                                                               |
| ncomp                | Number of components to extract (rank-k). Default 2.                                                                                                                                                                                                                                                                                                   |
| preproc_x, preproc_y | Optional multivarious preprocessors (e.g., <code>center()</code> ). Defaults to <code>multivarious::pass()</code> (no-op).                                                                                                                                                                                                                             |
| svd_backend          | Character, one of "eigencore" (default) or "irlba" for the iterative SVD. This choice only matters for larger problems: whenever both X and Y have at most 64 columns after preprocessing, the operator materializes S densely and computes a direct <code>svd()</code> , ignoring <code>svd_backend</code> entirely (see <code>gplssvd_op()</code> ). |
| svd_opts             | List of options: <code>tol</code> for both backends and <code>maxitr</code> for irlba only. An incomplete eigencore solve raises an error of class <code>genpca_solver_nonconvergence</code> ; no unchecked fit is returned.                                                                                                                           |

constraints\_remedy      What to do with a metric that is not positive semi-definite: "error" (default), "ridge", "clip" or "identity"; repairs emit a genpca\_metric\_repaired warning. See [genpca\(\)](#).

verbose      Logical; print brief progress messages.

### Value

An object of class `c("genpls", "cross_projector", "projector")` with the same structure as `genpls()` returns (X-/Y-weights `vx/vy`, singular values `d`, generalized weights `p/q`, scores `fi/fj`, latent variables `lx/ly`, `ncomp`, and `backend`); see `?genpls` for the definition of each slot.

### References

Beaton, D. (2020). Generalized eigen, singular value, and partial least squares decompositions: The GSVD package. (Eqs. 10-14). [arXiv:2010.14734](#).

### See Also

[genpls\(\)](#)

### Examples

```
set.seed(1)
X <- matrix(rnorm(60 * 5), 60, 5)
Y <- matrix(rnorm(60 * 4), 60, 4)
fit <- genplsc(X, Y, ncomp = 2,
              preproc_x = multivarious::center(),
              preproc_y = multivarious::center())
fit$d
```

---

gmd\_clear\_cache

---

*Clear internal cache for matrix decompositions*


---

### Description

Clears the internal cache used by generalized matrix decomposition functions. This can be useful to free up memory or when working with different datasets.

### Usage

```
gmd_clear_cache()
```

### Value

Invisibly returns TRUE after clearing the cache.

## Examples

```
# Clear the internal cache
gmd_clear_cache()
```

---

gpca\_mle

---

*Experimental penalized-ML estimation of GPCA metrics*


---

## Description

Alternates between GPCA factor estimation and penalized maximum-likelihood updates of the row/column metric matrices ( $M$ ,  $A$ ) under a Gaussian matrix-normal error model with a low-rank mean. Each iteration performs three exact block minimizations of a single penalized objective:

1. Fit GPCA with current  $A$ ,  $M$ : the GMD theorem makes this the best rank- $n_{\text{comp}}$  fit in the  $(M, A)$  norm, so it exactly minimizes the residual term.
2. Update  $\Sigma_r = EAE^T/p + \lambda I$  (the exact block minimizer under the ridge penalty), set  $M = \text{solve}(\text{Sigma}_r)$ .
3. Update  $\Sigma_c = E^TME/n + \lambda I$  using the *updated*  $M$  (sequential flip-flop, Dutilleul 1999), set  $A = \text{solve}(\text{Sigma}_c)$ .

## Usage

```
gpca_mle(
  X,
  ncomp = min(dim(X)),
  max_iter = 20,
  lambda = 0.001,
  scale_fix = c("none", "trace", "det"),
  tol = 1e-04,
  method = "eigen",
  constraints_remedy = "error",
  preproc = multivarious::pass(),
  verbose = FALSE,
  ...
)
```

## Arguments

|                       |                                                                                                                                                                                                                                                                                                                                                                                          |
|-----------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>X</code>        | Numeric matrix ( $n \times p$ ).                                                                                                                                                                                                                                                                                                                                                         |
| <code>ncomp</code>    | Rank to extract at each GPCA step.                                                                                                                                                                                                                                                                                                                                                       |
| <code>max_iter</code> | Maximum outer alternations (default 20).                                                                                                                                                                                                                                                                                                                                                 |
| <code>lambda</code>   | Ridge penalty weight (default 1e-3). Part of the objective (MAP interpretation), not just a numerical safeguard: it shrinks both covariances toward a multiple of the identity and pins the row/column scale split during iteration. Must be non-negative; with <code>lambda = 0</code> the objective loses strict convexity in the scale direction and covariances may become singular. |

|                    |                                                                                                                                                                                                                                                                                                                                                                                                                                                               |
|--------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| scale_fix          | Optional post-hoc reparameterization of the $c * \Sigma_r$ , $\Sigma_c / c$ split at exit. One of "none" (default: keep the penalized optimum), "trace" (row covariance scaled to mean diagonal 1) or "det" (row covariance scaled to determinant 1). Applied as a joint reciprocal rescale, so the fitted covariance $\Sigma_r \otimes \Sigma_c$ and the unpenalized likelihood are unchanged, but the penalized objective generally decreases; see Details. |
| tol                | Relative tolerance on successive penalized log-likelihood change (default 1e-4) for early stopping.                                                                                                                                                                                                                                                                                                                                                           |
| method             | GPCA method passed to genpca (defaults to "eigen").                                                                                                                                                                                                                                                                                                                                                                                                           |
| constraints_remedy | Passed to genpca; defaults to "error". The learned metrics are inverses of positive definite matrices, so no repair fires in practice.                                                                                                                                                                                                                                                                                                                        |
| preproc            | Pre-processing transformer; defaults to <code>multivarious::pass()</code> .                                                                                                                                                                                                                                                                                                                                                                                   |
| verbose            | Logical; if TRUE, prints iteration diagnostics.                                                                                                                                                                                                                                                                                                                                                                                                               |
| ...                | Additional arguments forwarded to genpca.                                                                                                                                                                                                                                                                                                                                                                                                                     |

## Details

The objective is the matrix-normal log-likelihood with a low-rank mean and an inverse-Wishart-style ridge penalty  $\lambda (p \operatorname{tr} \Sigma_r^{-1} + n \operatorname{tr} \Sigma_c^{-1})$  (a MAP estimate). Because every block update is an exact minimizer of this one objective, `loglik_path` is monotone non-decreasing up to numerical noise. The penalty also resolves the  $c \Sigma_r$ ,  $\Sigma_c / c$  scale indeterminacy, so the converged metrics are the penalized optimum and no rescaling is needed (`scale_fix = "none"`, the default). `scale_fix = "trace"` or `"det"` additionally applies a joint reciprocal rescale at exit (row covariance normalized, factor absorbed into the column covariance). The unpenalized matrix-normal likelihood is invariant to that rescale, but the penalty  $p\lambda \operatorname{tr}(M) + n\lambda \operatorname{tr}(A)$  is not, so the rescaled metrics are no longer the penalized optimum; the returned `loglik` is always evaluated at the returned metrics and `loglik_rescale_delta` reports how far the rescale moved it. The algorithm stops when the relative change in the penalized log-likelihood falls below `tol` or `max_iter` is reached. Increase `lambda` or reduce `ncomp` if iterations become unstable. The objective is not identifiable with `lambda = 0`.

## Value

A list with elements `fit` (a `genpca` fit computed with the returned metrics), `A`, `M` (learned SPD metrics), `loglik` (the penalized log-likelihood evaluated at the returned `M`, `A` and `fit`), `loglik_unpenalized` (the same without the `lambda` penalty), `loglik_rescale_delta` (the change in the penalty contribution caused solely by reciprocal metric rescaling; exactly zero for `scale_fix = "none"`), `loglik_refit_delta` (the remaining change from the last path value to `loglik`, including final refitting and numerical objective reevaluation), and `loglik_path` (the penalized log-likelihood after each outer iteration; monotone non-decreasing up to numerical noise, since every block update exactly minimizes the shared penalized objective). Values omit additive constants and include the `lambda` penalty, so they are comparable across iterations and across runs with the same `lambda`, but not across different `lambda` values.

## References

Dutilleul, P. (1999). *The MLE algorithm for the matrix normal distribution*. Journal of Statistical Computation and Simulation, 64(2), 105-123.

## Examples

```
if (requireNamespace("multivarious", quietly = TRUE)) {
  set.seed(123)
  X <- matrix(rnorm(40), 8, 5)
  res <- gpca_mle(X, ncomp = 2, max_iter = 5, lambda = 1e-3,
                 scale_fix = "trace", verbose = FALSE)
  # Learned metrics are SPD and match dimensions
  dim(res$A); dim(res$M)
  res$loglik_path
}
```

---

gplssvd\_op

---

*Generalized PLS-SVD via Implicit Operator (memory-safe)*


---

## Description

Compute the top- $k$  singular triplets of  $S = Xe'Ye$  without materializing the whitened matrices  $Xe = Mx^{1/2}XWx^{1/2}$ ,  $Ye = My^{1/2}YWy^{1/2}$  when doing so would densify sparse data. When the whitening is sparsity-preserving (identity/diagonal metrics) or the data are dense, the whitened blocks are precomputed once so each matrix-vector product in the iterative SVD costs two multiplies.

## Usage

```
gplssvd_op(
  X,
  Y,
  XLW = NULL,
  YLW = NULL,
  XRW = NULL,
  YRW = NULL,
  k = 2,
  center = FALSE,
  scale = FALSE,
  svd_backend = c("eigencore", "irlba", "RSpectra"),
  svd_opts = list(tol = 1e-07, maxitr = 1000),
  constraints_remedy = c("error", "ridge", "clip", "identity")
)
```

**Arguments**

|                           |                                                                                                                                                                                                                                                                                                                                                                                                         |
|---------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>X</b>                  | n x I matrix (numeric or Matrix)                                                                                                                                                                                                                                                                                                                                                                        |
| <b>Y</b>                  | n x J matrix (numeric or Matrix)                                                                                                                                                                                                                                                                                                                                                                        |
| <b>XLW</b>                | Row metric for X (M_X): NULL/identity, numeric length-n, diagonalMatrix, or PSD Matrix                                                                                                                                                                                                                                                                                                                  |
| <b>YLW</b>                | Row metric for Y (M_Y)                                                                                                                                                                                                                                                                                                                                                                                  |
| <b>XRW</b>                | Column metric for X (W_X)                                                                                                                                                                                                                                                                                                                                                                               |
| <b>YRW</b>                | Column metric for Y (W_Y)                                                                                                                                                                                                                                                                                                                                                                               |
| <b>k</b>                  | Number of components. If k exceeds $\min(\text{ncol}(X), \text{ncol}(Y))$ , a warning is issued and k is silently truncated to that maximum.                                                                                                                                                                                                                                                            |
| <b>center, scale</b>      | Logical; pre-center/scale columns of X, Y before metrics                                                                                                                                                                                                                                                                                                                                                |
| <b>svd_backend</b>        | One of "eigencore" (default) or "irlba"; "RSpectra" is accepted as a deprecated alias of "eigencore". Ignored whenever both $\text{ncol}(X) \leq 64$ and $\text{ncol}(Y) \leq 64$ , in which case S is materialized densely and solved with <code>base::svd()</code> . Outside that fallback, the irlba backend requires both column counts to be at least six; use "eigencore" for narrower operators. |
| <b>svd_opts</b>           | List of options for the backend: <code>tol</code> (both backends) and <code>maxitr</code> (irlba only; the eigencore partial SVD has no iteration cap). An incomplete eigencore solve raises <code>genpca_solver_nonconvergence</code> ; try a less stringent <code>tol</code> if the requested accuracy cannot be reached.                                                                             |
| <b>constraints_remedy</b> | What to do with a metric that is not positive semi-definite: "error" (default), "ridge", "clip" or "identity"; repairs emit a <code>genpca_metric_repaired</code> warning. See <a href="#">genpca()</a> .                                                                                                                                                                                               |

**Details**

Naming map: this function names its metrics XLW/YLW (left/row weights) and XRW/YRW (right/column weights); these correspond to  $M_X/M_Y$  (row metrics) and  $A_X/A_Y$  (column metrics) in `genpca()`'s and `genpls()`'s M/A convention.

**Value**

A list with elements:

- d** Length-k numeric vector of singular values of  $S = Xe'Ye$ .
- u** I x k matrix; left singular vectors of S (orthonormal in the Euclidean metric).
- v** J x k matrix; right singular vectors of S (orthonormal in the Euclidean metric).
- p** I x k matrix of generalized X-weights,  $p = W_X^{-1/2}u$ .
- q** J x k matrix of generalized Y-weights,  $q = W_Y^{-1/2}v$ .
- fi** I x k matrix of X-variable scores,  $F_i = W_X p D$  (columns of p rescaled by the singular values).
- fj** J x k matrix of Y-variable scores,  $F_j = W_Y q D$ .
- lx** N x k matrix of X row latent variables,  $L_x = M_X^{1/2} X W_X p$ .
- ly** N x k matrix of Y row latent variables,  $L_y = M_Y^{1/2} Y W_Y q$ .

**k** Integer; number of components actually returned (may be less than the requested k if it exceeded  $\min(I, J)$ ).

**dims** A list `list(N, I, J)` with the row count N and column counts  $I = \text{ncol}(X)$ ,  $J = \text{ncol}(Y)$ .

**center** A list `list(X, Y)` of the length-I / length-J column means subtracted from  $X/Y$  (all zero when `center = FALSE`).

**scale** A list `list(X, Y)` of the length-I / length-J column scale factors divided out of  $X/Y$  (all one when `scale = FALSE`).

## References

Beaton, D. (2020). Generalized eigen, singular value, and partial least squares decompositions: The GSVD package. arXiv:2010.14734.

Abdi, H. (2007). Partial least square regression PLS-Regression. In N. Salkind (Ed.), *Encyclopedia of Measurement and Statistics*. Thousand Oaks, CA: Sage.

## Examples

```
set.seed(1)
X <- matrix(rnorm(40 * 6), 40, 6)
Y <- matrix(rnorm(40 * 4), 40, 4)
op <- gplssvd_op(X, Y, k = 2, center = TRUE)
round(op$d, 3)
```

---

mnpca\_mrl

---

*Matrix-Normal PCA via Maximum Regularized Likelihood*


---

## Description

Fits a rank-ncomp matrix factorization under matrix-normal noise with sparse row/column precision matrices:

$$Y = XW^T + E, \quad E \sim \mathcal{MN}(0, \Omega, \Sigma)$$

using alternating block updates:

1. Alternating least squares updates for X, W with fixed precisions ( $\text{Theta\_row} = \Omega^{-1}$ ,  $\text{Theta\_col} = \Sigma^{-1}$ ).
2. Graphical-lasso style precision updates for  $\text{Theta\_row}$  and  $\text{Theta\_col}$  using ADMM, warm starts, and optional block screening.

The precision updates are solved on a residual-free scatter surrogate (rather than the exact residual  $E = Y - XW^T$ ), so the two block updates do not jointly minimize a single shared objective at each step; as a result the reported `objective_path` is a convergence diagnostic, not a monotone objective trace (see Details and the `objective_path` return value).

**Usage**

```

mnpca_mrl(
  Y,
  ncomp = min(dim(Y)),
  lambda_row = 0.05,
  lambda_col = 0.05,
  max_outer = 25,
  max_inner = 5,
  tol = 1e-04,
  eps_ridge = 1e-08,
  jitter = 1e-06,
  center = TRUE,
  update_precisions = TRUE,
  warm_start = TRUE,
  gl_maxit = 200,
  gl_tol = 1e-04,
  gl_rho = 1,
  penalize_diagonal = FALSE,
  block_screen = TRUE,
  scale_fix = c("trace", "none"),
  sparsify_threshold = 1e-08,
  as_sparse_precision = TRUE,
  verbose = FALSE
)

```

**Arguments**

|                   |                                                                          |
|-------------------|--------------------------------------------------------------------------|
| Y                 | Numeric matrix (n x p).                                                  |
| ncomp             | Target rank (r).                                                         |
| lambda_row        | L1 penalty for row precision (Theta_row).                                |
| lambda_col        | L1 penalty for column precision (Theta_col).                             |
| max_outer         | Maximum number of outer BCD iterations.                                  |
| max_inner         | Maximum ALS steps per outer iteration.                                   |
| tol               | Relative tolerance used for ALS and objective convergence checks.        |
| eps_ridge         | Ridge added to small r x r normal-equation systems.                      |
| jitter            | Small diagonal jitter used in covariance/precision updates.              |
| center            | Logical; center columns of Y before fitting.                             |
| update_precisions | Logical; if FALSE, keeps identity precisions and runs weighted ALS only. |
| warm_start        | Logical; warm start precision updates from previous iterate.             |
| gl_maxit          | Maximum ADMM iterations per graphical-lasso subproblem.                  |
| gl_tol            | ADMM convergence tolerance for graphical-lasso subproblems.              |
| gl_rho            | ADMM augmented Lagrangian parameter.                                     |

|                     |                                                                                                               |
|---------------------|---------------------------------------------------------------------------------------------------------------|
| penalize_diagonal   | Logical; whether to penalize diagonal precision entries in L1 term. Default FALSE.                            |
| block_screen        | Logical; use thresholded connected components to solve precision subproblems blockwise.                       |
| scale_fix           | One of "trace" or "none". "trace" normalizes each precision to mean diagonal 1 after updates.                 |
| sparsify_threshold  | Off-diagonal magnitude threshold used to set tiny precision entries to zero after each graphical-lasso solve. |
| as_sparse_precision | Logical; store precisions as sparse matrices when many entries are zero.                                      |
| verbose             | Logical; print iteration diagnostics.                                                                         |

### Details

The covariance updates use low-rank correction identities and avoid explicit construction of  $E = Y - XW^T$ .

This function returns a plain S3 list of class "mnpca\_mrl", not a **multivarious** `bi_projector/cross_projector`; the `scores()/components()/reconstruct()` generics used elsewhere in this package do not apply here. Use the returned `$X`, `$W`, and `$fitted` fields directly.

This implementation follows the maximum regularized likelihood (MRL) formulation of MN-PCA, combining low-rank factor updates with sparse precision estimation in row and column spaces, via alternating surrogate updates rather than joint minimization of a single objective at every step.

The main optimization target is:

$$\frac{1}{2} \text{tr} \left( \Theta_c (Y - XW^T)^T \Theta_r (Y - XW^T) \right) - \frac{p}{2} \log |\Theta_r| - \frac{n}{2} \log |\Theta_c| + n\lambda_r \|\Theta_r\|_1 + p\lambda_c \|\Theta_c\|_1$$

with  $\Theta_r \succ 0$ ,  $\Theta_c \succ 0$ . When `update_precisions = FALSE`, the method reduces to weighted low-rank approximation with fixed identity precisions.

Because the ALS factor updates and the graphical-lasso precision updates are solved against different surrogates of this target (the precision step uses a residual-free scatter matrix rather than the exact residual), the outer alternation is best understood as alternating surrogate updates with a relative-objective-change convergence heuristic, not classical block coordinate descent on a single monotone objective.

### Value

An object of class "mnpca\_mrl" with components:

**X, W** Estimated low-rank factors ( $n \times r$ ,  $p \times r$ ).

**Theta\_row, Theta\_col** Estimated row/column precision matrices.

**fitted** Reconstructed matrix on input scale.

**fitted\_centered** Reconstructed centered matrix used in optimization.

**residual\_centered** Centered residual matrix.

**objective\_path** Objective value evaluated after each outer iteration's block updates, before any trace rescaling of the precisions. Because the factor updates (ALS) and precision updates (graphical lasso on a residual-free scatter surrogate) target different surrogates rather than a single shared objective, this path is a convergence heuristic and is **not guaranteed to be monotone**.

**iterations** Number of outer iterations used.

**converged** Logical; TRUE if the relative change in `objective_path` fell below `tol` before `max_outer` was reached. This is a convergence heuristic based on relative objective change, not a guarantee of a local optimum.

**center** Column centering vector (or NULL).

**call** Matched call.

## References

Zhang, C., Gai, K., & Zhang, S. (2024). *Matrix normal PCA for interpretable dimension reduction and graphical noise modeling*. Pattern Recognition, 154, 110591. doi:10.1016/j.patcog.2024.110591

Friedman, J., Hastie, T., & Tibshirani, R. (2008). *Sparse inverse covariance estimation with the graphical lasso*. Biostatistics, 9(3), 432-441.

## Examples

```
set.seed(123)
n <- 20; p <- 12; r <- 3
Y <- matrix(rnorm(n * p), n, p)
fit <- mnpca_mrl(
  Y,
  ncomp = r,
  lambda_row = 0.08,
  lambda_col = 0.08,
  max_outer = 6,
  max_inner = 6,
  verbose = FALSE
)
dim(fit$X)
dim(fit$W)
length(fit$objective_path)
```

---

print.sfpca

---

*Print an sfpca fit*


---

## Description

Prints a one-line summary of an `sfpca` object: number of components, dimensions, and singular values.

**Usage**

```
## S3 method for class 'sfpca'
print(x, ...)
```

**Arguments**

|     |                  |
|-----|------------------|
| x   | An sfpca object. |
| ... | Ignored.         |

**Value**

x, invisibly.

---

|                    |                                           |
|--------------------|-------------------------------------------|
| reconstruct.genpca | <i>Reconstruct data from a genpca fit</i> |
|--------------------|-------------------------------------------|

---

**Description**

Reconstructs (an approximation of) the original data from a genpca fit as `ou[, comp] %*% diag(d[comp]) %*% t(ov[, comp])`, followed by the inverse of the preprocessing transform. With all components and full rank this recovers the original data.

**Usage**

```
## S3 method for class 'genpca'
reconstruct(
  x,
  comp = 1:multivarious::ncomp(x),
  rowind = NULL,
  colind = NULL,
  ...
)
```

**Arguments**

|        |                                                                                                                                           |
|--------|-------------------------------------------------------------------------------------------------------------------------------------------|
| x      | A genpca object.                                                                                                                          |
| comp   | Integer vector of components to use (default: all).                                                                                       |
| rowind | Optional integer vector of rows to reconstruct (default: all).                                                                            |
| colind | Optional integer vector of columns to reconstruct (default: all). The inverse preprocessing transform is applied to the selected columns. |
| ...    | Ignored.                                                                                                                                  |

**Value**

A numeric matrix of dimension `length(rowind) x length(colind)`.

**See Also**

[genpca\(\)](#), [truncate.genpca\(\)](#)

**Examples**

```
X <- matrix(rnorm(60), 15, 4)
fit <- genpca(X, ncomp = 4, preproc = multivarious::center())
max(abs(reconstruct(fit) - X)) # ~ 0 at full rank
```

---

|                   |                                           |
|-------------------|-------------------------------------------|
| reconstruct.sfpca | <i>Reconstruct data from an sfpca fit</i> |
|-------------------|-------------------------------------------|

---

**Description**

Reconstructs the rank-K [sfpc](#) model as  $UDV'$ , using the stored (non-orthogonal) factors directly.

**Usage**

```
## S3 method for class 'sfpc'
reconstruct(
  x,
  comp = seq_len(multivarious::ncomp(x)),
  rowind = NULL,
  colind = NULL,
  ...
)
```

**Arguments**

|        |                                                                   |
|--------|-------------------------------------------------------------------|
| x      | An sfpc object.                                                   |
| comp   | Integer vector of components to use (default: all).               |
| rowind | Optional integer vector of rows to reconstruct (default: all).    |
| colind | Optional integer vector of columns to reconstruct (default: all). |
| ...    | Ignored.                                                          |

**Details**

sfpc components are Euclidean unit vectors but are **not** mutually orthogonal, so  $V'V \neq I$ . The inherited `reconstruct.bi_projector()` method reconstructs through the Moore-Penrose pseudoinverse of the loadings (`scores \%*\% pinv(V)`), which for non-orthogonal  $V$  does **not** return the rank-comp model  $UDV'$  that `sfpc()` actually fits and deflates with. This method instead computes `scores(x)[rowind, comp] \%*\% t(components(x)[colind, comp])`, i.e.  $UDV'$  restricted to the requested rows/columns/components. `sfpc()` does no preprocessing, so no inverse transform is applied.

**Value**

A numeric matrix of dimension  $\text{length}(\text{rowind}) \times \text{length}(\text{colind})$ , the rank-length(comp) reconstruction  $UDV'$  using `sfpca`'s stored non-orthogonal factors.

**See Also**

[sfpca\(\)](#)

---

|                            |                                          |
|----------------------------|------------------------------------------|
| <code>repair_metric</code> | <i>Repair a metric matrix explicitly</i> |
|----------------------------|------------------------------------------|

---

**Description**

Returns a positive (semi)definite version of `A` together with a diagnostic report of what was done. This is the explicit counterpart of the `constraints_remedy` argument of [genpca\(\)](#): nothing in the package repairs a metric silently, and this function lets you inspect the repair before using the result.

**Usage**

```
repair_metric(
  A,
  method = c("ridge", "clip", "identity"),
  rtol = .metric_rtol_default(),
  name = "A",
  diag_maxn = 2000L
)
```

**Arguments**

|                        |                                                                                                                                                                                                                                                                                                                                                                                                      |
|------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>A</code>         | A square symmetric matrix (base matrix or <code>Matrix</code> ). Asymmetry beyond round-off is an error (see the relative asymmetry test in <code>symmetrize_or_stop()</code> ); it is not something a PSD repair should hide.                                                                                                                                                                       |
| <code>method</code>    | "ridge" adds a diagonal loading that makes the matrix positive definite (Gershgorin-based shift, falling back to <code>Matrix::nearPD()</code> for small dense matrices); "clip" projects onto the PSD cone by zeroing negative eigenvalues (dense eigendecomposition; refuses large sparse input); "identity" replaces an indefinite matrix by the identity (the report still describes the input). |
| <code>rtol</code>      | Relative tolerance: eigenvalues above $-\text{rtol} \times \text{scale}(A)$ count as non-negative for "ridge" and "identity", which then return the matrix unchanged. "clip" always removes negative eigenvalues, regardless of this tolerance (up to reconstruction roundoff). Default <code>sqrt(.Machine\$double.eps)</code> .                                                                    |
| <code>name</code>      | Label used in messages.                                                                                                                                                                                                                                                                                                                                                                              |
| <code>diag_maxn</code> | Largest dimension for which the report computes the full spectrum (minimum eigenvalue, rank, condition number); above it an iterative minimum eigenvalue estimate and a Gershgorin bound are reported, with rank and condition number unavailable.                                                                                                                                                   |

**Value**

The repaired matrix (a Matrix), with attribute "repair\_report" of class "metric\_repair\_report":  
a list with name, method, changed, n, min\_eigenvalue\_before, min\_eigenvalue\_after, gershgorin\_bound\_before, shift (diagonal loading added by "ridge", NA for "clip"), rank, condition\_number and rtol.

**See Also**

[genpca\(\)](#) (argument constraints\_remedy)

**Examples**

```
A <- matrix(c(1, 2, 2, 1), 2)      # eigenvalues 3 and -1
B <- repair_metric(A, method = "ridge")
attr(B, "repair_report")
C <- repair_metric(A, method = "clip")
eigen(as.matrix(C))$values
```

---

rpls

Regularised / Generalised Partial Least Squares (RPLS / GPLS)

---

**Description**

Implements the algorithm of Allen *et al.* (2013) for supervised dimension-reduction with optional sparsity ( $\ell_1$ ) or ridge ( $\ell_2$ ) penalties **and** the generalised extension that operates in a user-supplied quadratic form  $Q$ .

**Usage**

```
rpls(
  X,
  Y,
  K = 2,
  lambda = 0.1,
  penalty = c("l1", "ridge"),
  Q = NULL,
  nonneg = FALSE,
  preproc_x = multivarious::pass(),
  preproc_y = multivarious::pass(),
  tol = 1e-06,
  maxiter = 200,
  verbose = FALSE,
  ...
)
```

**Arguments**

|                             |                                                                                                                                                                      |
|-----------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>X</b>                    | Numeric matrix ( $n \times p$ ) — predictors.                                                                                                                        |
| <b>Y</b>                    | Numeric matrix ( $n \times q$ ) — responses.                                                                                                                         |
| <b>K</b>                    | Integer, number of latent factors to extract. Default 2.                                                                                                             |
| <b>lambda</b>               | Scalar or length-K numeric vector of penalties.                                                                                                                      |
| <b>penalty</b>              | Either "l1" (lasso) or "ridge".                                                                                                                                      |
| <b>Q</b>                    | Optional positive-(semi)definite $p \times p$ matrix inducing <i>generalised</i> PLS. NULL means identity.                                                           |
| <b>nonneg</b>               | Logical, force non-negative loadings when penalty = "l1". Note: This option is currently ignored when penalty = "ridge".                                             |
| <b>preproc_x, preproc_y</b> | Optional <b>multivarious</b> preprocessing objects (see <a href="#">fit_transform</a> ). By default they pass the data through unchanged using <code>pass()</code> . |
| <b>tol</b>                  | Relative tolerance for the inner iterations convergence check. Default 1e-6.                                                                                         |
| <b>maxiter</b>              | Maximum number of inner iterations per component. Default 200.                                                                                                       |
| <b>verbose</b>              | Logical; print progress messages during component extraction. Default FALSE.                                                                                         |
| <b>...</b>                  | Further arguments (e.g., custom stopping criteria if implemented) are stored in the returned object (they are not used by <code>fit_rpls</code> ).                   |

**Details**

Unlike `genpls()`, which handles separate row and column metrics ( $M_x, A_x, M_y, A_y$ ) with a Gram–Schmidt orthogonalisation step, `rpls()` uses a single metric  $Q$  and the simpler penalised updates of Allen et al.

**Value**

An object of class `c("rpls", "cross_projector", "projector")` with at least the elements

**vx**  $p \times K$  matrix of X-loadings.

**vy**  $q \times K$  matrix of Y-loadings.

**ncomp** Number of components actually extracted (may be  $< K$ ).

**penalty** Penalty type used ("l1" or "ridge").

**lambda** The lambda value(s) used.

**tol** The convergence tolerance used.

**maxiter** The maximum number of inner iterations used per component.

**nonneg** Logical flag for the non-negativity constraint on l1.

**Q\_used** Logical; TRUE if a custom  $Q$  metric was supplied to induce generalised RPLS, FALSE for standard (identity-metric) RPLS. Note the field is named `Q_used`, not `Q`: the metric matrix itself is not retained on the returned object.

**verbose** The verbose flag used.

**preproc\_x, preproc\_y** Pre-processing transforms used.

rpls objects store no row (X-)scores: the factors  $z_k$  are computed internally during fitting to drive deflation and are then discarded, so use `multivarious::project()` on `X` to obtain scores for any rows of interest.

The object supports `predict()`, `project()`, `transfer()`, `coef()` and other **multivarious** generics.

## Method

The routine follows Algorithm 1 of Allen *et al.* (2013, *Stat. Anal. Data Min.*, 6 : 302–314) — see the paper for details. Briefly, with  $C = X^\top Y$  the cross-product of the (preprocessed) blocks, each component maximises

$$\max_{u,v} v^\top Q C u - \lambda P(v)$$

with  $Q = I_p$  for standard RPLS. The alternating updates are:  $u \leftarrow C^\top Q v / \|C^\top Q v\|_2$ , then a penalised (possibly non-negative) regression for  $v$ , normalised in the  $Q$ -norm.

## References

Allen, G. I., Peterson, C., Vannucci, M., & Maletić-Savatić, M. (2013). *Regularized Partial Least Squares with an Application to NMR Spectroscopy*. **Statistical Analysis and Data Mining**, 6(4), 302-314. DOI:10.1002/sam.11169.

## Examples

```
# Generate sample data
set.seed(123)
n <- 50
p <- 20
q <- 10
X <- matrix(rnorm(n * p), n, p)
Y <- X[, 1:5] %*% matrix(rnorm(5 * q), 5, q) + matrix(rnorm(n * q), n, q)

# Fit regularized PLS with L1 penalty
fit_l1 <- rpls(X, Y, K = 3, lambda = 0.1, penalty = "l1")
print(fit_l1)

# Fit regularized PLS with ridge penalty
fit_ridge <- rpls(X, Y, K = 3, lambda = 0.1, penalty = "ridge")
print(fit_ridge)
```

---

sfpca

*Sparse and Functional Principal Components Analysis (SFPCA) with Spatial Coordinates*

---

## Description

Performs Sparse and Functional PCA on a data matrix, allowing for both sparsity and smoothness in the estimated principal components. Penalty parameters left NULL are selected automatically (see Details). The spatial smoothness penalty is constructed based on provided spatial coordinates.

**Usage**

```

sfpca(
  X,
  K,
  spat_cds,
  lambda_u = NULL,
  lambda_v = NULL,
  alpha_u = NULL,
  alpha_v = NULL,
  Omega_u = NULL,
  penalty_u = "l1",
  penalty_v = "l1",
  nlambda = 10,
  lambda_min_ratio = 0.01,
  knn = min(6, ncol(X) - 1),
  max_iter = 100,
  tol = 1e-06,
  verbose = FALSE,
  uthresh = NULL,
  vthresh = NULL
)

```

**Arguments**

|          |                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |
|----------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| X        | A numeric data matrix of dimensions n (observations/time points) by p (variables/space).                                                                                                                                                                                                                                                                                                                                                                           |
| K        | The number of principal components to estimate.                                                                                                                                                                                                                                                                                                                                                                                                                    |
| spat_cds | A matrix of spatial coordinates for each column of X (variables). Each row corresponds to a spatial dimension (e.g., x, y, z), and each column corresponds to a variable. Note the orientation: this is dimensions x variables, so <code>ncol(spat_cds)</code> must equal <code>ncol(X)</code> – the transpose of the layout a coordinate data frame usually has. For a one-dimensional axis (a spectrum, a transect) pass <code>matrix(coords, nrow = 1)</code> . |
| lambda_u | Sparsity penalty parameter for u. If NULL, selected per component by BIC along a regularization path (see Details).                                                                                                                                                                                                                                                                                                                                                |
| lambda_v | Sparsity penalty parameter for v. If NULL, selected per component by BIC along a regularization path (see Details).                                                                                                                                                                                                                                                                                                                                                |
| alpha_u  | Smoothness penalty parameter for u. If NULL, defaults to $1 / \lambda_{\max}(\Omega_u)$ (see Details).                                                                                                                                                                                                                                                                                                                                                             |
| alpha_v  | Smoothness penalty parameter for v. If NULL, defaults to $1 / \lambda_{\max}(\Omega_v)$ (see Details).                                                                                                                                                                                                                                                                                                                                                             |
| Omega_u  | A positive semi-definite matrix for smoothness penalty on u. If NULL, defaults to second differences penalty (sparse matrix). Unlike <code>Omega_u</code> , there is no corresponding <code>Omega_v</code> argument: the column-side smoothness penalty is always built internally from <code>spat_cds</code> (via <code>knn</code> ); supplying a custom <code>Omega_v</code> is not currently supported.                                                         |

|                  |                                                                                                                         |
|------------------|-------------------------------------------------------------------------------------------------------------------------|
| penalty_u        | The penalty function for u. Either "l1" (lasso, the default) or "scad".                                                 |
| penalty_v        | The penalty function for v. Either "l1" (lasso, the default) or "scad".                                                 |
| nlambda          | Number of values on the regularization path used for BIC selection of lambda_u/lambda_v when they are NULL. Default 10. |
| lambda_min_ratio | Smallest path value as a fraction of the closed-form lambda_max, on a log-spaced grid. Default 1e-2.                    |
| knn              | Number of nearest neighbours for constructing Omega_v. Default min(6, ncol(X) - 1).                                     |
| max_iter         | Maximum number of iterations for the alternating optimization. Default 100.                                             |
| tol              | Tolerance for convergence of the rank-1 objective. Default 1e-6.                                                        |
| verbose          | Logical; if TRUE, prints progress messages.                                                                             |
| uthresh          | Deprecated and ignored; lambda_u is now selected by BIC.                                                                |
| vthresh          | Deprecated and ignored; lambda_v is now selected by BIC.                                                                |

## Details

Each rank-1 problem is solved by alternating solves of the penalized quadratic subproblems (via C++ coordinate descent) followed by rescaling onto the smoothness-metric ball, in the constraint form of Allen & Weylandt (2019). For the convex "l1" penalty with subproblems solved to tolerance (the internal exact\_inner = TRUE path, used by the monotonicity test) the objective is monotonically non-decreasing; the default inexact path tightens the inner tolerance to a floor before it may declare convergence, reproducing the same terminal iterates but without an every-iteration monotonicity guarantee (it may also stop at max\_iter).

When lambda\_u or lambda\_v is NULL it is selected per component by a BIC-style criterion along a regularization path. For the convex "l1" penalty lambda\_max = max(abs(b)) is, in closed form, the smallest value whose subproblem solution is exactly zero (at x = 0 the S x term vanishes, so the KKT condition |b\_j| <= lambda does not depend on S); where b is the matrix-vector product with the other factor fixed at the SVD initializer. For the non-convex "scad" penalty the same value anchors the path but is not a global-optimality threshold. nlambda values are laid log-spaced down to lambda\_min\_ratio \* lambda\_max, coordinate descent is warm-started along the path, and the value minimizing log(RSS / (n p)) + df \* log(n p) / (n p) is chosen, with df the support size of the solution and RSS the one-sided rank-1 residual sum of squares with the opposite factor held fixed (a selection heuristic, not the BIC of the fully alternated rank-1 model). The all-zero solution (at lambda\_max) is a legitimate candidate: if no rank-1 structure justifies its degrees of freedom, the component is returned as exactly zero with d = 0.

When alpha\_u or alpha\_v is NULL it defaults to 1 / lambda\_max(Omega), so the roughest direction of the smoothness penalty is weighted exactly as strongly as the identity term. This makes the default invariant to the scaling of Omega and bounds the condition number of every subproblem system I + alpha \* Omega by 2.

## Value

An object of class c("sfpc", "bi\_projector") from the **multivarious** framework. Use multivarious::scores() for the sample scores (UD), multivarious::components() for the sparse loadings V, multivarious::sdev() for d\_k, and multivarious::reconstruct() for the rank-K approximation. ov (like components())

holds the sparse right factors  $V$ ; `ou` holds the left factors  $U$ . The selected penalty parameters are stored as `lambda_u`, `lambda_v`, `alpha_u`, and `alpha_v`. For backward compatibility the pre-0.1 list fields `$d` (singular values) and `$u` (left factors) remain readable but emit a deprecation warning; use `sdev()` and `scores()/ou` instead.

**Important:** unlike `genpca()`, the columns of  $U$  (`ou`) and  $V$  (`ov`) are Euclidean unit-norm but are **not** mutually orthogonal across components – `sfpca()` extracts each rank-1 term from a constraint-form subproblem rather than a joint SVD, so  $U'U \neq I$  and  $V'V \neq I$  in general. Consequently `multivarious::sdev()` here is *not* the singular values of  $X$ ; it is the per-component captured covariance  $d_k = u_k' X_k v_k$ , where  $X_k$  is the matrix after the preceding components have been deflated out (so the identity holds against  $X$  itself only for  $k = 1$ ). This non-orthogonality is also why `reconstruct()` for "sfpca" objects uses the stored  $U$ ,  $d$ ,  $V$  factors directly ( $U D V'$ ) rather than SVD-based identities such as the Moore-Penrose pseudoinverse of the loadings, which would not reproduce the fitted model for non-orthogonal  $V$  (see `reconstruct.sfpca()`).

## References

Allen, G. I., & Weylandt, M. (2019). Sparse and functional principal components analysis. In *2019 IEEE Data Science Workshop (DSW)* (pp. 11-16). doi:10.1109/DSW.2019.8755778. Also available as [arXiv:1309.2895](https://arxiv.org/abs/1309.2895), first posted in 2013 and revised through 2019; the preprint and the DSW paper are the same work, which is why both years appear in the literature.

## See Also

[genpca\(\)](#) for the shared **multivarious** verbs; `multivarious::bi_projector`.

## Examples

```
library(Matrix)
set.seed(123)
# Smooth temporal factor, sparse spatial factor
n <- 100 # Number of time points
p <- 50  # Number of spatial locations
u <- sin(seq(0, 2 * pi, length.out = n))
v <- c(rnorm(10), rep(0, p - 10))
X <- 8 * tcrossprod(u / sqrt(sum(u^2)), v / sqrt(sum(v^2))) +
  matrix(rnorm(n * p, sd = 0.2), n, p)
spat_cds <- matrix(runif(p * 3), nrow = 3, ncol = p) # 3D coordinates
result <- sfpca(X, K = 1, spat_cds = spat_cds)
multivarious::sdev(result) # captured covariance (BIC-tuned)
sum(multivarious::components(result) != 0) # sparse spatial loading
```

---

transfer.cross\_projector

*Compatibility wrapper for multivarious::transfer*

---

## Description

Provides a transfer method that accepts source/target arguments used in the tests. The method forwards to `multivarious`'s transfer method which expects from/to.

**Usage**

```
## S3 method for class 'cross_projector'
transfer(x, new_data, from = NULL, to = NULL, opts = list(), ...)
```

**Arguments**

|                       |                                                                                                                                                                                                          |
|-----------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>x</code>        | A <code>cross_projector</code> object.                                                                                                                                                                   |
| <code>new_data</code> | New data to transfer.                                                                                                                                                                                    |
| <code>from</code>     | Source space ("X" or "Y").                                                                                                                                                                               |
| <code>to</code>       | Target space ("X" or "Y").                                                                                                                                                                               |
| <code>opts</code>     | Options list passed to <code>multivarious::transfer</code> .                                                                                                                                             |
| <code>...</code>      | Additional arguments. Legacy parameters <code>source</code> and <code>target</code> are accepted here for backwards compatibility but are deprecated; use <code>from</code> and <code>to</code> instead. |

**Value**

Matrix with transferred data.

**Examples**

```
# Generate sample data
set.seed(123)
n <- 50
X <- matrix(rnorm(n * 10), n, 10)
Y <- matrix(rnorm(n * 8), n, 8)

# Create a cross projector using rpls
fit <- rpls(X, Y, K = 2)

# Transfer new X data to Y space
new_X <- matrix(rnorm(10 * 10), 10, 10)
transferred <- transfer(fit, new_X, from = "X", to = "Y")
```

---

truncate

*Truncate a projection to fewer components*


---

**Description**

Re-exported from **multivarious**. See [truncate](#) for details.

**Usage**

```
truncate(x, ncomp)
```

**Arguments**

|       |                                |
|-------|--------------------------------|
| x     | A projection object            |
| ncomp | Number of components to retain |

**Value**

Truncated projection object

---

|                 |                                                  |
|-----------------|--------------------------------------------------|
| truncate.genpca | <i>Truncate a genpca fit to fewer components</i> |
|-----------------|--------------------------------------------------|

---

**Description**

Returns a new genpca object retaining only the first ncomp components. All component-indexed slots (v, s, sdev, ov, ou, u, propv, cumv) are sliced consistently; the preprocessing object and constraint matrices are carried over unchanged.

**Usage**

```
## S3 method for class 'genpca'
truncate(x, ncomp)
```

**Arguments**

|       |                                                                              |
|-------|------------------------------------------------------------------------------|
| x     | A genpca object.                                                             |
| ncomp | Number of components to retain (a positive integer no larger than ncomp(x)). |

**Value**

A genpca object with ncomp components.

**See Also**

[genpca\(\)](#), [reconstruct.genpca\(\)](#)

**Examples**

```
X <- matrix(rnorm(60), 15, 4)
fit <- genpca(X, ncomp = 4)
fit2 <- truncate(fit, 2)
multivarious::ncomp(fit2)
```

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