GNU Gama 1.9.01e

Adjustment of geodetic networks
Edition 1.9.01e (15 April 2006)

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1 Introduction

GNU Gama is a project dedicated to adjustment of *geodetic networks*. It was formerly inspired by Fortran system Geodet/PC (1990) designed by Frantisek Charamza. The GNU Gama project has been started at the department of mapping and cartography, faculty of Civil Engineering, Czech Technical University in Prague (CTU) about 1998 and its name is an acronym for *geodesy and mapping*. It was presented to wider public for the first time at FIG Working Week 2000 in Prague and then at FIG Workshop and Seminar at HUT Helsinki in 2001.

`http://www.gnu.org/software/gama/`

GNU Gama is released under GNU General Public Licence and is based on a C++ library of geodetic classes and functions and a small C++ template matrix library `matvec`. For parsing XML documents GNU Gama calls `expat` parser version 1.1 written by James Clark. The `expat` parser is not part of the GNU Gama project, but is used with GNU Gama.

GNU Gama currently contains two main development branches. Stable branch of the project is limited to network adjustment in a local coordinate system and its functionality is represented by a command line program `gama-local`. Program `gama-local` adjusts geodetic (free) networks of observed distances, directions, angles, height differences, 3D vectors and observed coordinates (coordinates with given variance-covariance matrix).

In the unstable development branch the adjustment model is based on geocentric coordinate system (adjustment model on ellipsoid). Its functionality is represented by command line program `gama-g3`. The stable branch is frozen but is being updated to be compatible with the model of adjustment in geocentric coordinates.

Project Rocinante is the GNU Gama GUI for the adjustment in local coordinate systems (stable branch; program `roci-local`) and is based on the Qt graphical library. Rocinante is documented separatedly, for more information see

`http://roci.sourceforge.net/`

Compiled executables of program `roci-local` corresponding to stable version 1.8 for Linux and Win32 can be downloaded from

`http://sourceforge.net/projects/roci/`

1.1 Download

The latest GNU Gama stable version can be found on in the subdirectory `/gnu/gama/` on your favourite GNU mirror or from the CVS as the branch `1.8`. See our project page at savannah for more information.

Development branch version `1.9.01e` is available from the CVS on the main trunk or from the alpha GNU FTP server `ftp://alpha.gnu.org/gnu/gama/`. Archive of previous versions (both stable and unstable) is available at `http://gama.fsv.cvut.cz/gama/archive/`.

To get an anonymous read-only access to the CVS repository for the latest GNU Gama unstable source, submit the following commands

```
cvs -d:pserver:anoncvs@subversions.gnu.org:/cvsroot/gama login
cvs -z3 -d:pserver:anoncvs@subversions.gnu.org:/cvsroot/gama co gama
```

or

```
cvs -z3 -d:pserver:anoncvs@subversions.gnu.org:/cvsroot/gama
         co -r 1.8 gama
```

for the stable branch 1.8.

To download from CVS only the matrix/vector C++ template library `matvec` run

```
cvs -z3 -d:pserver:anoncvs@subversions.gnu.org:/cvsroot/gama
         co gama/matvec
```

1.1.1 Examples

The collection of sample networks is available as a separete CVS module. To checkout the `gama-local` examples from CVS use the command

```
cvs -z3 -d:pserver:anoncvs@subversions.gnu.org:/cvsroot/gama
         co examples
```

1.2 Install

GNU Gama is developed and tested under Debian GNU/Linux (http://www.debian.org/). You can compile Gama easily yourself if you download the sources. If `expat` XML parser is installed on your system, change to the directory of Gama project and issue the following commands at the shell prompt:

```
$ make dep
$ make
$ make example
```

If for any reason `expat` is not installed on your system, you can still compile and build Gama with old version `expat 1.1` that is shipped with Gama

```
$ make dep-expat-1.1
$ make
```

To compile GNU Gama under Windows (command-line tool `gama-local`) start a DOS window, change to the directory `gamaprog/win32-borland` and run `make` (for Borland free compiler `bcc32`) or alternatively to the directory `gamaprog/win32-msvc` and run Microsoft `nmake`.

1.3 Program gama-local

Program `gama-local` is a simple command line tool for adjustment of geodetic *free networks.* It is available for GNU Linux (the main platform on which project GNU Gama is being developped) or for MS Windows (tested with Borland compiler from Borland free command line tools and with Microsoft Visual C++ compiler; support for Windows platform is currently limited to maintaing compatibility with the two mentioned compilers).

Program `gama-local` reads input data in XML format (Chapter 2 [XML input data format for gama-local], page 5) and prints adjustment results into ASCII text file. If output file name is not given, input file name with extension `.txt` is used. If run without arguments `gama-local` prints a short help

```
$ ./gama-local
```

```
Adjustment of local geodetic network        version: 1.8.01 / GNU g++
************************************
http://www.gnu.org/software/gama/

Usage: gama-local [options]  input.xml  [ output. ]

Options:

--algorithm  svd | gso | cholesky
--language   en | ca | cz | du | fi | fr | hu | ru | ua
--encoding   utf-8 | iso-8859-2 | iso-8859-2-flat | cp-1250 | cp-1251
--angles     400 | 360
--latitude   <latitude>
--ellipsoid  <ellipsoid name>
--xmlout     adjustment_results.xml
--version
--help
```

Program `gama-local` version is followed by information on compiler used to build the program (apart from GNU `g++` compiler, two other possibilities are `bcc` and `msc` for Borland and Microsoft compilers respectively, when build under Microsoft Windows).

Option `--algorithm` enables to select numerical method used for solution of the adjustment. Implicitly is used Singular Value Decomposition (`svd`), alternatively user can decide for block matrix algorithm GSO by Frantisek Charamza, based on Gram-Schmidt ortogonalization. In both these cases, project equations are solved directly without forming *normal equations*. Third possibility is to select Cholesky decomposition of semidefinite matrix of normal equations (`cholesky`).

Option `--language` selects language used in output protocol. For example, if run with option `--language cz`, `gama-local` prints output results in Czech languague using UTF-8 encoding. Implicit value is `en` for output in English.

Option `--encoding` enables to change inplicit UTF-8 output encoding to iso-8859-2 (latin-2), iso-8859-2-flat (latin-2 without diacritics), cp-1250 (MS-EE encoding) cp-12251 (Russian encoding).

Option `--angles` selects angular units to be used in output.

1.4 Program `gama-g3`

There is no documentation for unstable development branch program `gama-g3` yet ...

1.5 Contributors

The following persons (in chronological order) have made contributions to GNU Gama project: Ales Cepek, Jiri Vesely, Petr Doubrava, Jan Pytel, Chuck Ghilani, Dan Haggman, Mauri Vaisanen, John Dedrum, Jim Sutherland, Zoltan Faludi, Diego Berge, Boris Pihtin, Stephane Kaloustian, Anton Horpynich and Claudio Fontana.

1.6 Reporting bugs

Undoubtedly there are numerous bugs remaining, both in the C++ source code and in the documentation. If you find a bug in either, please send a bug report to

bug-gama@gnu.org

We will try to be as quick as possible in fixing the bugs and redistributing the fixes. If you prefere, you can always write directly to Ales Cepek.

2 XML input data format for gama-local

The input data format for a local geodetic network adjustment (program gama-local) is defined in accordance with the definition of Extended Markup Language (XML) for description of structured data. The XML definition can be found at

http://www.w3.org/TR/REC-xml

Input data (points, observations and other related information) are described using XML start-end pair tags <xxx> and </xxx> and empty-element tags <xxx/>. The syntax of XML input format is defined in the Document Type Definition (DTD) at

http://www.gnu.org/software/gama/gama-xml.dtd

and can formally be validated independently on the program gama-local.

For parsing the XML input data, gama-local uses the XML parser Expat copyrighted by James Clark which is described at

http://www.jclark.com/xml/expat.html

Expat is subject to the Mozilla Public License (MPL), or may alternatively be used under the GNU General Public License (GPL) instead.

In the gama-local XML input, distances are given in metres, angular values in centigrades and their standard deviations (rms errors) in millimetres or centigrade seconds, respectively. Alternatively angular values in gama-local XML input can be given in degrees and seconds (see Section 2.1 [Angular units], page 5). At the end of this document an example of the gama-local XML input data object is given.

2.1 Angular units

Horizontal angles, directions and zenith angles in gama-local XML adjustment input are implicitly given in gons and their standard deviations and/or variances in centicentigons. Gon, also called centesimal grade and Neugrad (German for new grad), is 1/400-th of the circumference. For example

```
<direction  from="202" to="416" val="63.9347"  stdev="10.0" />
```

The same angular value (direction) can be expressed in degrees as

```
<direction  from="202" to="416" val="57-32-28.428"  stdev="3.24" />
```

In XML adjustment input degrees are coded as a single string, where degrees (57), minutes (32) and seconds (28.428) are separated by dashes (-) with optional leading sign. Spaces are not allowed inside the string. Gons and degrees may be mixed in a single XML document but one should be careful to supply the information on standard deviations and/or covariances in the proper corresponding units.

Internaly gama-local works with gons but output can be transformed to degrees using the option --angles 360.

2.2 Prolog

XML documents may, and should, begin with an XML declaration that specifies the version of XML being used (*prolog*). In the case of gama-local, the XML input data are followed by the XML document type declaration:

```
<?xml version="1.0" ?>
<!DOCTYPE gama-xml
  SYSTEM "http://www.gnu.org/software/gama/gama-xml.dtd">
```

2.3 Tags <gama-xml> and <network>

A pair tag `<gama-xml>` contains a single pair tag `<network>` that contains the network definition. The definition of the network is composed of three sections:

- `<description>` of the network (annotation or comments),
- network `<parameters />` and
- `<points-observations>` section.

The sections `<description>` and `<parameters />` are optional, the section `<points-observations>` is mandatory. These three sections may be presented in any order and may be repeated several times (in such a case, the corresponding sections are linked together by the software).

The pair tag `<network>` has two optional attributes `axes-xy` and `angles`. These attributes are used to describe orientation of the `xy` orthogonal coordinate system axes and the orientation of the observed angles and/or directions.

- `axes-xy="ne"` orientation of axes `x` and `y`; value `ne` implies that axis `x` is oriented north and axis `y` is oriented east. Acceptable values are `ne`, `sw`, `es`, `wn` for right-handed coordinate systems and `en`, `nw`, `se`, `ws` for left-oriented coordinate systems (default value is `ne`).
- `angles="right-handed"` defines clockwise observed angles and/or directions, value `left-handed` defines counterclockwise observed angles and/or directions (default value is `right-handed`).

Many geodetic systems are right handed with `x` axis oriented north, `y` axis oriented east and clockwise angular observations. Example of right-handed orthogonal system with different axes orientation is coordinate system *Krovak* used in the Czech Republic where the axes `x` and `y` are oriented south and west respectively. Apart from right handed systems the program `gama-local` can adjust observations and coordinates in left-handed systems with counterclockwise angles and/or directions.

The current version does not support mixed systems with different orientation of axes and angles. If such a situation is detected, `gama-local` tries to recover from the unsupported system by simply internally changing the sign of all `y` coordinates (in the text output of adjustment results the sign of `y` coordinates is reverted back by `gama-local`).

Example

```
<gama-xml>
<network>
   <description> ... </description>
   <parameters ... />
   <points-observations> ... </points-observations>
</network>
</gama-xml>
```

It is planned in future versions of the program to allow more `<network>` tags (analysis of deformations etc.) and definitions of new tags.

2.4 Network description

The description of a geodetic network is enclosed in the start-end pair tags `<description>`. Text of the description is copied into the adjustment output and serves for easier identification of results. The text is not interpreted by the program, but it may be helpful for users.

Example

```
<description>
A short description of a geodetic network ...
</description>
```

2.5 Network parameters

The network parameters may be listed with the following optional attributes of an empty-element tag `<parameters />`

- `sigma-apr = "10"` value of a priori reference standard deviation—square root of reference variance (default value 10)
- `conf-pr = "0.95"` confidence probability used in statistical tests (dafault value 0.95)
- `tol-abs = "1000"` tollerance for identification of gross absolute terms in project equations (default value 1000 mm)
- `sigma-act = "apriori"` actual type of reference standard deviation use in statistical tests (`aposteriori | apriori`); default value is `apriori`
- `update-constrained-coordinates = "no"` enables user to control if coordinates of constrained points are updated in iterative adjustment. If test on linerarization fails (see Section 3.9 [Linearization], page 32), Gama tries to improve approximate coordinates of adjusted points and repeats the whole adjustment. Coordinates of constrained points are implicitly not changed during iterations.

Values of the attributes must be given either in the double-quotes (`"..."`) or in the single quotes (`'...'`). There can be *white spaces* (spaces, tabs and new-line characters) between attribute names, values, and the *equal* sign.

Example

```
<parameters sigma-apr = "15"
            conf-pr   = '0.90'
            sigma-act = "apriori"
            update-constrained-coordinates = "no" />
```

2.6 Points and observations

The points and observations section is bounded by the pair tag `<points-observations>` and contains information about points, observed horizontal directions, angles, and horizontal distances, height differences, slope distances, zenith angles, observed vectors and control coordinates.

Optional attributes of the start tag `<points-observations>` allow for the definition of default values of standard deviations corresponding to observed directions, angles, and distances.

- `direction-stdev = "..."` defines the implicit value of observed direction (default value is not defined)
- `angle-stdev = "..."` defines the implicit value of observed angle (default value is not defined)
- `zenith-angle-stdev = "..."` defines the implicit value of observed zenith angle (default value is not defined)
- `distance-stdev = "..."` defines the implicit value of observed horizontal distance (default value is not defined)

Implicit values of standard deviations for the observed distances are calculated from the model with three constants a, b, and c according to the formula

$$a + bD^c,$$

where a is a constant part of the model and D is the observed distance in kilometres. If the constants b and/or c are not given, default values of $b = 0$ and $c = 1$ will be used.

Example

```
<points-observations direction-stdev = "10"
                     distance-stdev  = "5 3 1" >
   <!-- ... points and observation data ... -->
</points-observations>
```

2.7 Points

Points are described by the empty-element tags `<point/>` with the following attributes:

- `id = "..."` is the point identification attribute (mandatory); point identification is not limited to *numbers*; all printable characters can be used in identification.
- `x = "..."` specifies coordinate `x`
- `y = "..."` specifies coordinate `y`
- `z = "..."` specifies coordinate `z`, point height
- `fix = "..."` specifies coordinates that are fixed in adjustment; acceptable values are `xy`, `XY`, `z`, `Z`, `xyz`, `XYZ`, `xyZ` and `XYz`.
- `adj = "..."` specifies coordinates to be adjusted (unknown parameters in adjustment); acceptable values are `xy`, `XY`, `z`, `Z`, `xyz`, `XYZ`, `xyZ` and `XYz`.

With exception of the first attribute (point id), all other attributes are optional. Decimal numbers can be used as needed.

Control coordinates marked using the `fix` parameter are not changed in the adjustment. Uppercase and lowercase notation of coordinates with the `fix` parameter are interpreted the same. Corrections are applied to the unknown parameters identified by coordinates written in lowercase characters given in the `adj` parameter. When the coordinates are written using uppercase, they are interpreted as *constrained coordinates.* If coordinates are marked with both the `fix` and `adj`, the `fix` parameter will take precedence.

Constrained coordinates are used for the regularization of free networks. If the network is not free (fixed network), the *constrained* coordinates are interpreted as other unknown parameters. In classical free networks, the *constrained* points define the regularization constraint

$$\sum dx_i^2 + dy_i^2 = \min .$$

where dx and dy are adjusted coordinate corrections and the summation index i goes over all *constrained* points. In other words, the set of the *constrained* points defines the adjustment of the free network (its shape and size) with a simultaneous transformation to the approximate coordinates of selected points. Program `gama-local` allows the definition of constrained coordinates with 1D leveling networks, 2D and 3D local networks.

Example

```
<point id="1" y="644498.590" x="1054980.484" fix="xy"  />
<point id="2" y="643654.101" x="1054933.801" adj="XY" />
<point id="403" adj="xy" />
```

2.8 Set of observations

The pair tag `<obs>` groups together a set of observations which are somehow related. A typical example is a set of directions and distances observed from one stand-point. An observation section contains a set of

- horizontal directions `<direction ... />`
- horizontal distances `<distance ... />`
- horizontal angles `<angle ... />`
- slope distances `<s-distance ... />`
- zenith angles `<z-angle ... />`
- height differences `<dh />`

The band variance-covariance matrix of directions, distances, and angles listed in one `<obs>` section may be supplied using a `<cov-mat>` pair tag with attributes `dim` (dimension) and `band` (bandwidth). The band-width of the diagonal matrix is equal to 0 and a fully-populated variance-covariance matrix has a bandwidth of `dim-1`.

Observation variances and covariances (i.e. an upper-symmetric part of the band-matrix) are written row by row between `<cov-mat>` and `</cov-mat>` tags. If present, the dimension of the variance-covariance matrix must agree with the number of observations.

The following example of variance-covariance matrix with dimension 6 and bandwidth 2 (two nonzero codiagonals and three zero codiagonals)

$$\begin{pmatrix} 1.1 & 0.1 & 0.2 & 0 & 0 & 0 \\ 0.1 & 1.2 & 0.3 & 0.4 & 0 & 0 \\ 0.2 & 0.3 & 1.3 & 0.5 & 0.6 & 0 \\ 0 & 0.4 & 0.5 & 1.4 & 0.7 & 0.8 \\ 0 & 0 & 0.6 & 0.7 & 1.5 & 0.9 \\ 0 & 0 & 0 & 0.8 & 0.9 & 1.6 \end{pmatrix}$$

is coded in XML as

```
<cov-mat dim="6" band="2">
   1.1  0.1  0.2
        1.2  0.3  0.4
             1.3  0.5  0.6
                  1.4  0.7  0.8
                       1.5  0.9
                            1.6
</cov-mat>
```

If two or more sets of directions with different orientations are observed from a stand-point, they must be placed in different `<obs>` sections. The value of an orientation angle can be explicitly stated with an attribute `orientation="..."`. Normally, it is more convenient to let the program calculate approximate values of orientations needed for the adjustment. If directions are present, then the attribute `station` must be defined.

Optional attribute `from_dh="..."` enables to enter implicit height of instrument for all observations within the `<obs>` pair tag.

Observed distances are expressed in meters, their standard deviations in millimeters. Observed directions and angles are expressed in centigrades (400) and their standard deviations in centigrade seconds.

Height differences can be entered in the `<obs>` or `<height-differences>` section. If entered in the `<obs>` section, the `dist="..."` parameter is ignored (Section 2.14 [Height differences], page 13).

Example

```
<obs from="418">
   <direction  to=  "2" val="0.0000"   stdev="10.0" />
   <direction  to="416" val="63.9347"  stdev="10.0" />
   <direction  to="420" val="336.3190" stdev="10.0" />
   <distance   to="420" val="246.594"  stdev="5.0"  />
</obs>

<obs from="418">
   <direction  to=  "2" val="0.0000"   />
   <direction  to="416" val="63.9347"  />
   <direction  to="420" val="336.3190" />
   <distance   to="420" val="246.594"  />

   <cov-mat dim="4" band="0">
```

```
        100.00 100.00 100.00 25.00
      </cov-mat>
    </obs>
```

2.9 Directions

Directions are expressed with the following attributes in an empty-element tag `<direction />`

- `to = "..."` target point identification
- `val = "..."` observed direction; see Section 2.1 [Angular units], page 5
- `stdev = "..."` standard deviation (optional)
- `from_dh = "..."` instrument height (optional)
- `to_dh = "..."` reflector/target height (optional)

The standard deviation is an optional attribute. However since all observations in the adjustment must have their weights defined, the standard deviation must be given either explicitly with the attribute `stdev="..."` or implicitly with `<points-observation direction-stdev="..." >` or with a variance-covariance matrix for the given observation set. A similar approach applies to all the observations (distances, angles, etc.)

Example

```
<direction  to=  "2" val="0.0000"  stdev="10.0" />
<direction  to="416" val="63.9347" />
```

2.10 Horizontal distances

Distances are written using an empty-element tag `<distance />` with attributes

- `from = "..."` standpoint identification
- `to = "..."` target identification
- `val = "..."` observed horizontal distance
- `stdev = "..."` standard deviation of observed horizontal distance (optional)
- `from_dh = "..."` instrument height (optional)
- `to_dh = "..."` reflector/target height (optional)

Contrary to directions, distances in an observation set (`<obs>`) do not need to share a common stand-point. An example is set of distances observed from several stand-points with a common variance-covariance matrix.

Example

```
<distance from = "2"  to = "1" val = "659.184" />
<distance to ="422" val="228.207"  stdev="5.0"  />
<distance to ="408" val="568.341" />
```

2.11 Angles

Observed angles are expressed with the following attributes of an empty-element tag `<angle />`

- `from = "..."` standpoint identification (optional)
- `bs = "..."` backsight target identification
- `fs = "..."` foresight target identification
- `val = "..."` observed angle; see Section 2.1 [Angular units], page 5
- `stdev = "..."` standard deviation (optional)
- `from_dh = "..."` instrument height (optional)
- `bs_dh = "..."` backsight reflector/target height (optional)
- `fs_dh = "..."` foresight reflector/target height (optional)

Similar to distance observations, one observation set may group angles observed from several standpoints.

Example

```
<angle from="433" bs="422" fs="402" val="128.6548" stdev="14.1"/>
<angle from="433" bs="422" fs="402" val="128.6548" />
<angle bs="422" fs="402" val="128.6548" stdev="14.1"/>
<angle bs="422" fs="402" val="128.6548"/>
```

2.12 Slope distances

Slope distances (space distances) are written using an empty-element tag `<s-distance />` with attributes

- `from = "..."` standpoint identification (optional)
- `to = "..."` target identification
- `val = "..."` observed slope distance
- `stdev = "..."` standard deviation of observed slope distance (optional)
- `from_dh = "..."` instrument height (optional)
- `to_dh = "..."` reflector/target height (optional)

Similar to horizontal distances, one observation set may group slope distances observed from several standpoints.

Example

```
<s-distance from = "2"  to = "1" val = "658.824" />
<s-distance to ="422" val="648.618"  stdev="5.0"  />
<s-distance to ="408" val="482.578" />
```

2.13 Zenith angles

Zenith angles are written using an empty-element tag `<z-angle />` with the following attributes

- `from = "..."` standpoint identification (optional)
- `to = "..."` target identification
- `val = "..."` observed zenith angle; see Section 2.1 [Angular units], page 5
- `stdev = "..."` standard deviation of observed zenith angle (optional)
- `from_dh = "..."` instrument height (optional)
- `to_dh = "..."` reflector/target height (optional)

Similar to horizontal distances, one observation set may group zenith angles observed from several standpoints.

Example

```
<z-angle from = "2"  to = "1" val = "79.6548" />
<z-angle to ="422" val="85.4890"  stdev="5.0"  />
<z-angle to ="408" val="95.7319" />
```

2.14 Height differences

A set of observed leveling height differences is described using the start-end tag `<height-differences>` without parameters. The `<height-differences>` tag can contain a series of height differences (at least one) and can optionally be supplied with a variance-covariance matrix. Single height differences are defined with empty tags `<dh />` having the following attributes:

- `from = "..."` standpoint identification
- `to = "..."` target identification
- `val = "..."` observed leveling height difference
- `stdev = "..."` standard deviation of levellin elevation and
- `dist = "..."` distance of leveling section (in meters)

If the value of standard deviation is not present and length of leveling section (in kilometres) is defined, the value of standard deviation is computed from the formula

$$m_{dh} = m_0 \sqrt{D_{km}}.$$

If the value of standard deviation of the height difference is defined, information on leveling section length is ignored. A third possibility is to define a common variance-covariance matrix for all elevations in the set.

Example

```
<height-differences>
  <dh from="A" to="B" val=" 25.42" dist="18.1" />
  <dh from="B" to="C" val=" 10.34" dist=" 9.4" />
```

```
  <dh from="C" to="A" val="-35.20" dist="14.2" />
  <dh from="B" to="D" val="-15.54" dist="17.6" />
  <dh from="D" to="E" val=" 21.32" dist="13.5" />
  <dh from="E" to="C" val="  4.82" dist=" 9.9" />
  <dh from="E" to="A" val="-31.02" dist="13.8" />
  <dh from="C" to="D" val="-26.11" dist="14.0" />
</height-differences>
```

2.15 Control coordinates

Control (known) coordinates are described by the start-end pair tag `<coordinates>`. A series of points with known coordinates can be defined using the `<point />` tag. The variance-covariance matrix for the entire set of points can be created with a single `<cov-mat>` tag. In the `<point />` tags, a point identification (ID) and its coordinates (x, y and z) must be listed. Although the order of the `<point />` tag attributes is irrelevant in the corresponding variance-covariance matrix, the expected order of the coordinates is x, y and z (the horizontal coordinates x, y, or the height z might be missing, but not both). The type of the points may be defined either directly within the `<coordinates>` tag or outside of it.

Example

```
<coordinates>
   <point id="1" x="100.00" y="100.00" />
   <point id="2" z="200.00" y="200.00" x="200.00" />
   <point id="3" z="300.00" />
   <cov-mat dim="6" band="5" >
       ...  <!-- covariances for 1x 1y 2x 2y 2z 3z -->
   </cov-mat>
</coordinates>
```

2.16 Coordinate differences (vectors)

Observed coordinate differences describe relative positions of station pairs (vectors). Contrary to the observed coordinates, the variance-covariance matrix of the coordinate differences always describes all three elements of the 3D vectors.

Optional attributes of empty element tag `<vec>` for describing instrument and/or target height are

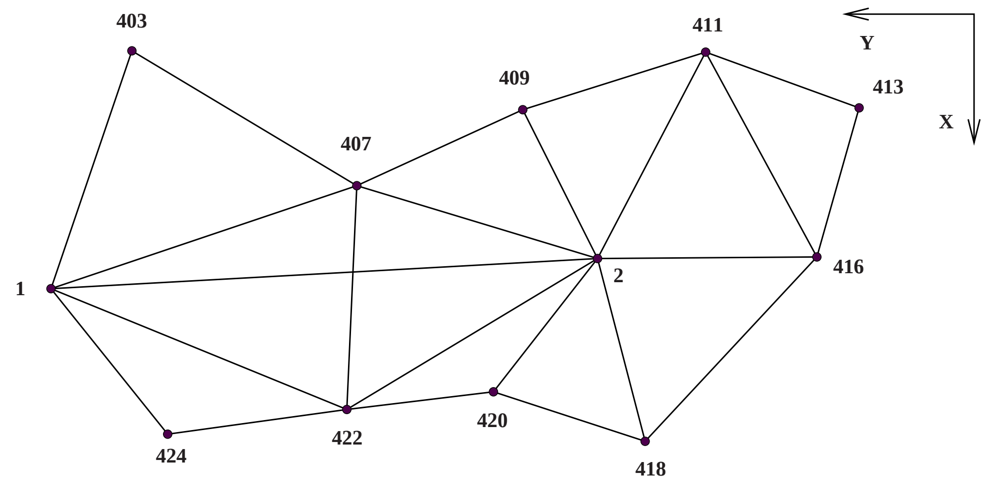
- `from_dh = "..."` instrument height
- `to_dh = "..."` target height

Example

```
<vectors>
   <vec from="id1" to="id2" dx="..." dy="..." dz="..." />
   <vec from="id2" to="id3" dx="..." dy="..." dz="..." />
   ...
   <cov-mat dim="..." band="..." >
       ..
   </cov-mat>
</vectors>
```

2.17 Example of local geodetic network

The XML input data format should be now reasonably clear from the following sample geodetic network. This example is taken from user's guide to Geodet/PC by Frantisek Charamza.




```
<?xml version="1.0" ?>
<!DOCTYPE gama-xml
  SYSTEM "http://www.gnu.org/software/gama/gama-xml.dtd">

<gama-xml>
<network axes-xy="sw">

<description>
XML input stream of points and observation data for program Gama
<!-- this is a XML comment -->
</description>

<!-- parameters are expressed with empty-element tag -->

<parameters sigma-act = "aposteriori" />

<points-observations>

<!-- fixed point, constrained point -->

<point id="1" y="644498.590" x="1054980.484" fix="xy" />
<point id="2" y="643654.101" x="1054933.801" adj="XY" />

<!-- computed / adjusted points -->
```

```
<point id="403" adj="xy" />
<point id="407" adj="xy" />
<point id="409" adj="xy" />
<point id="411" adj="xy" />
<point id="413" adj="xy" />
<point id="416" adj="xy" />
<point id="418" adj="xy" />
<point id="420" adj="xy" />
<point id="422" adj="xy" />
<point id="424" adj="xy" />

<obs from="1">
     <direction  to=  "2" val=  "0.0000" stdev="10.0" />
     <direction  to="422" val= "28.2057" stdev="10.0" />
     <direction  to="424" val= "60.4906" stdev="10.0" />
     <direction  to="403" val="324.3662" stdev="10.0" />
     <direction  to="407" val="382.8182" stdev="10.0" />
     <distance   to=  "2" val= "845.777" stdev="5.0"  />
     <distance   to="422" val= "493.793" stdev="5.0"  />
     <distance   to="424" val= "288.301" stdev="5.0"  />
     <distance   to="403" val= "388.536" stdev="5.0"  />
     <distance   to="407" val= "498.750" stdev="5.0"  />
</obs>

<obs from="2">
     <direction  to=  "1" val="0.0000"   stdev="10.0" />
     <direction  to="407" val="22.2376"  stdev="10.0" />
     <direction  to="409" val="73.8984"  stdev="10.0" />
     <direction  to="411" val="134.2090" stdev="10.0" />
     <direction  to="416" val="203.0706" stdev="10.0" />
     <direction  to="418" val="287.2951" stdev="10.0" />
     <direction  to="420" val="345.6928" stdev="10.0" />
     <direction  to="422" val="368.9908" stdev="10.0" />
     <distance   to="407" val="388.562"  stdev="5.0"  />
     <distance   to="409" val="257.498"  stdev="5.0"  />
     <distance   to="411" val="360.282"  stdev="5.0"  />
     <distance   to="416" val="338.919"  stdev="5.0"  />
     <distance   to="418" val="292.094"  stdev="5.0"  />
     <distance   to="420" val="261.408"  stdev="5.0"  />
     <distance   to="422" val="452.249"  stdev="5.0"  />
</obs>

<obs from="403">
     <direction  to=  "1" val="0.0000"   stdev="10.0" />
     <direction  to="407" val="313.5542" stdev="10.0" />
     <distance   to="407" val="405.403"  stdev="5.0"  />
</obs>
```

```
<obs from="407">
     <direction  to=  "1" val="0.0000"   stdev="10.0" />
     <direction  to="403" val="55.1013"  stdev="10.0" />
     <direction  to="409" val="193.3410" stdev="10.0" />
     <direction  to=  "2" val="239.4204" stdev="10.0" />
     <direction  to="422" val="323.5443" stdev="10.0" />
     <distance   to="409" val="281.997"  stdev="5.0"  />
     <distance   to="422" val="346.415"  stdev="5.0"  />
</obs>

<obs from="409">
     <direction  to=  "2" val="0.0000"   stdev="10.0" />
     <direction  to="407" val="102.2575" stdev="10.0" />
     <direction  to="411" val="310.1751" stdev="10.0" />
     <distance   to="411" val="296.281"  stdev="5.0" />
</obs>

<obs from="411">
     <direction  to=  "2" val="0.0000"   stdev="10.0" />
     <direction  to="409" val="49.8647"  stdev="10.0" />
     <direction  to="413" val="291.4953" stdev="10.0" />
     <direction  to="416" val="337.6667" stdev="10.0" />
     <distance   to="413" val="252.266"  stdev="5.0"  />
     <distance   to="416" val="360.449"  stdev="5.0"  />
</obs>

<obs from="413">
     <direction  to="411" val="0.0000"   stdev="10.0" />
     <direction  to="416" val="295.3582" stdev="10.0" />
     <distance   to="416" val="239.745"  stdev="5.0"  />
</obs>

<obs from="416">
     <direction  to=  "2" val="0.0000"   stdev="10.0" />
     <direction  to="411" val="68.8065"  stdev="10.0" />
     <direction  to="413" val="117.9922" stdev="10.0" />
     <direction  to="418" val="348.1606" stdev="10.0" />
     <distance   to="418" val="389.397"  stdev="5.0"  />
</obs>

<obs from="418">
     <direction  to=  "2" val="0.0000"   stdev="10.0" />
     <direction  to="416" val="63.9347"  stdev="10.0" />
     <direction  to="420" val="336.3190" stdev="10.0" />
     <distance   to="420" val="246.594"  stdev="5.0"  />
</obs>
```

```
<obs from="420">
     <direction  to=  "2" val="0.0000"   stdev="10.0" />
     <direction  to="418" val="77.9221"  stdev="10.0" />
     <direction  to="422" val="250.1804" stdev="10.0" />
     <distance   to="422" val="228.207"  stdev="5.0"  />
</obs>

<obs from="422">
     <direction  to=  "2" val="0.0000"   stdev="10.0" />
     <direction  to="420" val="26.8834"  stdev="10.0" />
     <direction  to="424" val="225.7964" stdev="10.0" />
     <direction  to=  "1" val="259.2124" stdev="10.0" />
     <direction  to="407" val="337.3724" stdev="10.0" />
     <distance   to="424" val="279.405"  stdev="5.0"  />
</obs>

<obs from="424">
     <direction  to=  "1" val="0.0000"   stdev="10.0" />
     <direction  to="422" val="134.2955" stdev="10.0" />
</obs>

</points-observations>

</network>
</gama-xml>
```

3 Network adjustment with `gama-local`

Adjustment of local geodetic network is a classical case of *adjustment of indirect observations.* After estimation of approximate values of unknown parameters (coordinates of points) and linearization of functions describing relations between observations and parameters we solve linear system of equations

$$\mathbf{A}\mathbf{x} = \mathbf{b} + \mathbf{v}, \tag{1}$$

where $\mathbf{A}$ is coefficient matrix, $\mathbf{b}$ is vector of absolute terms (right hand side) and $\mathbf{v}$ is vector of residuals. This system is (generally) overdetermined and we seek the solution $\mathbf{x}$ satisfying the basic criterion of Least Squares

$$\mathbf{v}'\mathbf{P}\mathbf{v} = \min, \tag{2}$$

where $\mathbf{P}$ is weight matrix. This criterion unambiguously defines the shape of adjusted network.

In the case of *free network* the system (1) is singular (matrix $\mathbf{A}$ has linearly dependent columns) and we have to define second regularization criterion

$$\sum_{i\in\Omega} x_i^2 = \min, \tag{3}$$

stating that at the same time we demand that the sum of squares corrections of selected parameters is minimal (corrections of unknown parameters with indexes from the set Ω). Geometrically this criterion is equivalent to adjustment of the network according to (2) with simultaneous transformatin to the selected set of fiducial points. This transformation does not change the shape of adjusted network.

Often it is advantageous to work with a *homogenized system,* ie. with the system of project equations in which coefficient of each row and absolute term are multiplied by square root of the weight of corresponding observation.

$$\tilde{\mathbf{A}}\mathbf{x} = \tilde{\mathbf{b}}, \tag{4}$$

where $\tilde{\mathbf{A}} = \mathbf{P}^{1/2}\mathbf{A}$, $\tilde{\mathbf{b}} = \mathbf{P}^{1/2}\mathbf{b}$. Symbol $\mathbf{P}^{1/2}$ denotes diagonal matrix of square roots of observation weights (or Cholesky decomposition of covariance matrix in the case of correlated observations). To criterion (2) corresponds in the case of homogenized system criterion

$$\tilde{\mathbf{v}}'\mathbf{v} = \min\,. \tag{5}$$

Normal equations are clearly equivalent for both systems.

$$(\mathbf{A}'\mathbf{P}\mathbf{A})\mathbf{x} = (\mathbf{A}'\mathbf{P}\mathbf{b}) \quad \equiv \quad (\tilde{\mathbf{A}}'\tilde{\mathbf{A}})\mathbf{x} = (\tilde{\mathbf{A}}\tilde{\mathbf{b}}).$$

Between weight coefficients of the original system (1) and homogenized system (4) are the following relations

$$\begin{aligned}
q_{x_i} &= \tilde{q}_{x_i}, \qquad i = 1,\ldots,n,\\
q_{L_j} &= \tilde{q}_{L_j}/p_j, \qquad j = 1,\ldots,m,\\
q_{v_k} &= \tilde{q}_{v_k}/p_k = (1-\tilde{q}_{L_k})/p_k = 1/p_k - q_{L_k}, \qquad k = 1,\ldots,m.
\end{aligned}$$

3.1 Approximate coordinates

For computation of coefficients in system (1) (ie. during linearization) we need, first of all, an estimate of approximate coordinates of points and approximate values of orientations of observed directions sets.

Approximate values of unknown parameters are usually not known and we have to compute them from the available observations. For approximate value of orientation program `gama-local` uses median of all estimates from the given set of directions to the points with known coordinates. Median is less sensitive to outliers than arithmetic mean which is normally used for approximate estimate of orientations

During the phase of computation of approximate coordinate of points, program `gama-local` walks through the list of computed points and for each point gathers all determining elements pointing to points with known or previously computed coordinates. Determining elements are

outer bearing (oriented half-line) starting from the point with known coordinates and pointing to the computed point

distance between given and computed points

inner angle with vertex in the computed point and arms intersecting given points

For all combinations of determining elements program `gama-local` computes intersections and estimates approximate coordinates as the median of all available solutions.

If at least one point was resolved while iterating through the list, the whole cycle is repeated.

If no more coordinates can be solved using intersections and points with unknown coordinates are remaining, program tries to compute coordinates of unresolved points in a local coordinates system and obtain their coordinates using similarity transformation. If a transformation succeeds to resolve coordinates at least one computed point and there are still some points without coordinates left, the whole process is repeated. Classes for computation of approximate coordinates have been written by Jiri Vesely.

If program `gama-local` fails to compute approximate coordinates of some of the network points, they are eliminated from the adjustment and they are listed in the output listing.

With the outlined strategy, program `gama-local` is able to estimate approximate coordinates in most of the cases we normally meet in surveying profession. Still there are cases in which the solution fails. One example is an inserted horizontal traverse with sets of observed direction on both ends but without a connecting observed distance. The solution of approximate coordinates can fail when there is a number of gross error for example resulting from confusion of point identifications but in normal situations, leaving computation of approximate coordinates on program `gama-local` is recommended.

Example

```
Computation of approximate coordinates of points
************************************************

Number of points with given coordinates:      2
Number of solved points                :      2
Number of observations                 :      4
```

```
-----------------------------------------------------
Successfully solved points             :      0
Remaining unsolved points              :      2


List of unresolved points
*************************
422
424
```

3.2 Gross absolute terms

One of parameters in XML input of program `gama-local` is tolerance `tol-abs` for detecting of gross absolute terms in project equations. Observations with outlying absolute terms are always excluded from adjustment.

For measured distances program tests difference between observed value d_i and distance computed from approximate coordinates d_0

$$|d_i - d_0| > \mathtt{tol} - \mathtt{abs},$$

for observed directions program `gama-local` tests transverse deviation corresponding to absolute term b_i from project equations (1)

$$|b_i| d_0 > \mathtt{tol} - \mathtt{abs}$$

and similarly for angles, program tests the greater of two deviations corresponding to left and right distances (left and right arm of the angle)

$$|b_i| \max\{d_{0_l}, d_{0_r}\} > \mathtt{tol} - \mathtt{abs}.$$

Default value of parameter `tol-abs` is 1000 mm.

Example

```
Outlying absolute terms in project equations
********************************************

   i   standpoint       target          observed     absolute
============================================= value ===== term ==

   2          103          104 dir.    301.087900       -9989.1


Observations with outlying absolute terms removed
```

3.3 Parameters of statistical analysis

Program `gama-local` uses two basic statistical parameters

- confidence probability P (default value is 95%, see parameter `conf-pr`) and
- actual type of reference standard deviation m_{0a} (parameter `typ-m0`).

Confidence probability determines significance level on which statistical tests of adjusted quatities are carried. Actual type of reference standard deviations m_{0a} specifies whether during statistical analysis we use a priori reference standard deviation m_0 or a posteriori estimate $m_0^{'}$.

We can choose only the type of actual reference standard deviation (m_0 or $m_0^{'}$) but not its value. The value corresponds to a priori given value of reference standard deviation or to the results of adjustment. On the type of actual reference standard deviation depends the choice of density functions of stochastic quantities in statistical analysis of the adjustment.

> **A priori reference standard deviation** m_0 is used in the cases when we know its value in advance and with sufficient reliability. Another situation when m_0 is used are networks with low number of degrees of freedom (poorly overdetermined systems) or when veen degrees of freedom is zero. Examples may be analysis of network models etc.
>
> **A posteri estimate of reference standard deviation** $m_0^{'}$ is used in cases when a priori value of reference standard deviation m_0 is not known and when degrees of freedom is sufficiently high and reliable for empirical estimate of $m_0^{'}$.

The standard deviation of an adjusted quantity θ is computed in dependece on the choice of actual type of reference standard deviation m_{0a} according to formula

$$m_{\theta_i} = m_{0a}\sqrt{q_{\theta_{ii}}},$$

where $q_{\theta_{ii}}$ is weight coefficient (cofactor) of the i-th adjusted unknown parameter (coordinate or orientation, $\theta = x_i$) or i-th adjusted observation (distance, direction, $\ldots, \theta = L_i$).

Apart from standard deviation m_θ, program `gama-local` computes for adjusted quantity θ its *confidence interval* (Θ_1, Θ_2) in which the real value Θ is located with probability P

$$P(\Theta_1 < \Theta < \Theta_2) = P,$$

$$\Theta_1 = \theta - k_p m_\theta, \qquad \Theta_2 = \theta + k_p m_\theta,$$

where coefficient k_p depends on confidence probability P and in the case of low number of degrees of freedom on the choice of actual type of reference standard deviation m_{0a}.

Coefficient k_p is computed for $m_{0a} = m_0$ as critical value of normal distribution for probability $\alpha/2$, for the case of choice $m_{0a} = m_0^{'}$ as critical value of Student distribution on confidence level $\alpha/2$ with τ degrees of freedom

$$k_p = \begin{cases} u_{\alpha/2} & \text{if } m_{0a} = m_0, \\ t_{\alpha/2,\tau} & \text{if } m_{0a} = m_0^{'}. \end{cases}$$

Similarly confidence ellipses for adjusted points are defined in the following text.

3.4 Test on the reference standard deviation

Null hypothesis $H_0 : m_0 = m_0^{'}$ is tested versus alternative hypothesis $H_1 : m_0 \neq m_0^{'}$. Test criterion is ratio of a posteriori estimate of reference standard deviation

$$m_0^{'} = \sqrt{\mathbf{v}'\mathbf{P}\mathbf{v}/\tau}$$

and a priori reference standard deviation m_0 (input data parameter `m0-apr`). For given significance level α lower and upper bounds of interval (L, U) are computed so, that if hypothesis H_0 is true, probabilities $P(m_0^{'}/m_0 \leq D)$ and $P(m_0^{'}/m_0 \geq H)$ are equal to $\alpha/2$. Lower and upper bounds of the interval are computed as

$$L = \sqrt{(\chi^2_{1-\alpha/2,\tau}/\tau)}, \qquad U = \sqrt{(\chi^2_{\alpha/2,\tau}/\tau)}.$$

Probability

$$P(L < m_0^{'}/m_0 < U) = \texttt{conf} - \texttt{pr}$$

is by default 95%, this corresponds to 5% confidence level test.

Exceeding the upper limit H of the confidence interval can be caused even by a single gross error (one outlying observation). Method of Least Squares is generally very sensitive to presence of outliers. Safely can be detected only one observation whose elimination leads to maximal decrease of a posteriori estimate of reference standard deviation

$$m_0^{''} = \sqrt{(\mathbf{v}'\mathbf{P}\mathbf{v} - \delta)/(\tau - 1)}, \qquad \delta = \max(v_i^2/q_{v_i}), \tag{6}$$

where

$$q_{v_i} = 1/p_i - q_{L_i} \tag{7}$$

is weight coefficient of i -th residual. If the set of observations contains only one gross error, the outlying observation is likely to be detected, but this can not be guaranteed.

In addition, program `gama-local` computes a posteriori estimate of reference standard deviation separately for horizontal distances and directions and/or angles after formula from

$$m'_{0t} = \sqrt{\sum \tilde{v}^2_{i_t} / \sum \tilde{q}_{v_{i_t}}}, \qquad t = d, s,$$

where symbol t denotes observed distances, directions and/or angles.

Example

```
m0  apriori  :    10.00
m0’ empirical:     9.64         [pvv] : 3.43560e+03

During statistical analysis we work

- with empirical standard deviation 9.64
- with confidence level             95 %

Ratio m0’ empirical / m0 apriori: 0.964
95 % interval (0.773, 1.227) contains value m0’/m0
```

```
m0'/m0 (distances): 0.997   m0'/m0 (directions): 0.943

Maximal decrease of m0''/m0 on elimination of one observation: 0.892

Maximal studentized residual 2.48 exceeds critical value 1.95
on significance level 5 % for observation #35
<distance from="407" to="422" val="346.415" stdev="5.0" />
```

3.5 Information on points

Program `gama-local` lists separately review of coordinates of fixed and adjusted points; adjusted *constrained* coordinates are marked with `*`; see equation (3). Adjusted coordinate standard deviations m_x and m_y , and values for computing confidence intervals are given in the listing of adjusted coordinates (Section 3.3 [Statistical analysis], page 24). In the review index i is the index of unknown x_i from the system of project equations (1) corresponding to the point coordinates x and y .

Example

```
Fixed points
************

      point         x              y
========================================

          1  1054980.484     644498.590
          2  1054933.801     643654.101


Adjusted coordinates
********************

  i        point    approximate  correction  adjusted    std.dev conf.i.
====================== value ====== [m] ====== value ========== [mm] ===

            422
  2           x    1055167.22747  -0.00510 1055167.22237     2.7     5.4
  3           y     644041.46119   0.00023  644041.46142     2.5     5.1

            424
  4           X * 1055205.41198  -0.00056 1055205.41142     3.1     6.3
  5           Y *  644318.24425  -0.00125  644318.24300     3.6     7.2
```

For adjusted points, program summarizes information on standard ellipses, confidence ellipses, mean square positional errors (m_p), mean coordinate errors (m_{xy}) and coefficients g characterizing position of approximate coordinates with regard to the confidence ellipse.

Example

```
Mean errors and parameters of error ellipses
********************************************

  point     mp      mxy      mean error ellipse  conf.err. ellipse   g
========== [mm] == [mm] ==== a [mm] b  alpha[g] ==== a' [mm] b' ========

    422     3.6     2.6     2.7     2.5   187.0     6.8     6.4     0.8
    424     4.7     3.4     3.7     2.9   131.8     9.5     7.4     0.2
    403     5.7     4.0     4.3     3.6    78.9    11.0     9.3     1.1
```

Mean square positional error m_p and mean coordinate error (m_{xy}) are computed as

$$m_p = \sqrt{m_y^2 + m_x^2}, \qquad m_{xy} = m_p/\sqrt{2},$$

where m_y^2 and m_x^2 are squares of standard deviations (variances) of adjusted points coordinates.

Semimajor and semiminor axes of standard ellipse are denoted as a and b in the listing, bearing of semimajor axis is denoted as α and they are computed from covariances of adjusted coordinates

$$a = \sqrt{\frac{1}{2}(\mathrm{cov}\ yy + \mathrm{cov}\ xx + c)}, \qquad b = \sqrt{\frac{1}{2}(\mathrm{cov}\ yy + \mathrm{cov}\ xx - c)},$$

$$c = \sqrt{(\mathrm{cov}\ xx - \mathrm{cov}\ yy)^2 + 4(\mathrm{cov}\ xy)^2},$$

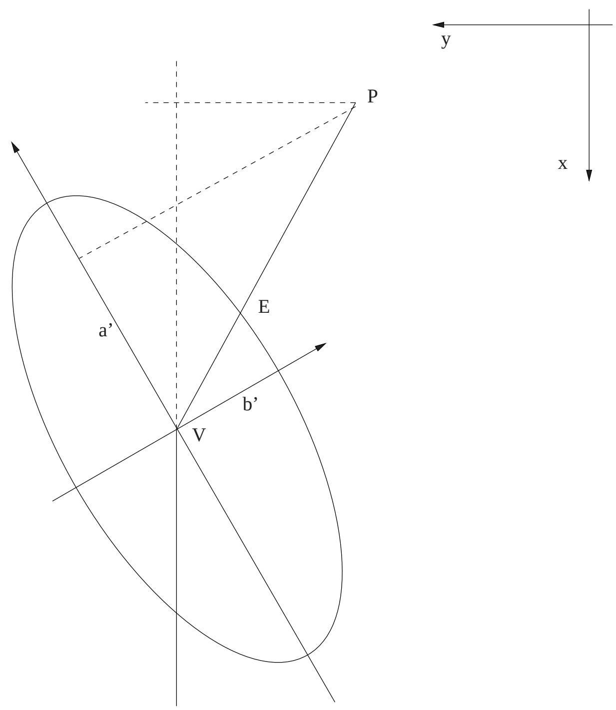
$$\tan 2\alpha = 2(\mathrm{cov}\ xy)/(\mathrm{cov}\ xx - \mathrm{cov}\ yy).$$

The angle α (the bearing of semimajor axis) is measured clockwise from X axis.

Probability that standard ellipse covers real position of a point is relatively low. For this reason program `gama-local` computes extra *confidence ellipse* for which the probability of covering real point position is equal to the given confidence probability. Both ellipsy are located in the same center, they share the same bearing of semimajor axes and they are similar. For lengths of their semi-axis holds

$$a' = k_p a, \qquad b' = k_p b,$$

where k_p is a coefficient computed for the given probability P as defined in Section 3.3 [Statistical analysis], page 24.



Position of approximate coordinates of an adjusted point with respect to its confidence ellipse is described by two points P and V where point P depicts approximate coordinates and V adjusted coordinates. Point E is the intersection of oriented half-line VP and the confidence ellipse. Coefficient g is defined as the ration of abscissae

$$g = \overline{VP}/\overline{VE}.$$

Three cases are possible

- $g < 1$ approximate coordinates of adjusted point are located inside the confidence ellipse
- $g = 1$ approximate coordinates of adjusted point are located on the confidence ellipse
- $g > 1$ approximate coordinates of adjusted point are outside the confidence ellipse

The coefficient g is calculated from formula

$$g = \sqrt{(a_0/a')^2 + (b_0/b')^2}$$

where

$$b_0 = \delta_y \cos\alpha - \delta_x \sin\alpha, \qquad a_0 = \delta_y \sin\alpha - \delta_x \cos\alpha$$

symbol δ is used for correction of approximate coordinates and α is bearing of confidence ellipse semimajor axis.

If network contains sets of observed directions, program writes information on corresponding adjusted orientations, standard deviations and confidence intervals. Index i is the same as in the case of adjusted coordinates the index of i -th adjusted unknown in the project equations.

Example

```
Adjusted bearings
*****************

  i   standpoint   approximate  correction  adjusted   std.dev conf.i.
=================== value [g] ==== [g] === value [g] ======= [cc] ===

  1            1    296.484371  -0.000917  296.483454      5.1    10.3
 10            2     96.484371   0.000708   96.485079      5.1    10.4
 21          403     20.850571  -0.001953   20.848618      8.8    17.7
```

3.6 Adjusted observations and residuals

In the review of adjusted observations program `gama-local` prints index of the observation, index of the row in matrix **A** in the system (1), identifications of standpoint and target point, type of the observation, its approximate and adjusted value, standard deviation and confidence interval.

Example

```
Adjusted observations
*********************

   i   standpoint   target         observed     adjusted std.dev conf.i.
===================================== value ==== [m|g] ====== [mm|cc] ==

   1            1        2 dis.   845.77700    845.77907     3.0     6.1
   2                   422 dir.   28.205700    28.205613     5.1    10.3
   3                   424 dir.   60.490600    60.491359     6.7    13.6
```

Review of residuals serves for analysis of observations and containts values of normalized or studentized residuals (depending on type of m_{0a} used) and three characteristics. Theese are coefficient `f` identifying weak network elements and estimates of real error of observation `e-obs` and real error of its adjusted value `e-adj`, see definition in the following text.

If normalized or studentized residual exceeds critical value for the given confidence probability, it is marked in the review with symbol `c` (critical) and maximal normalized or studentized residual is marked with symbol `m`.

Example

```
Residuals and analysis of observations
**************************************

   i standpoint   target        f[%]         v    |v'|     e-obs.  e-adj.
======================================== [mm|cc] =========== [mm|cc] ===

   1          1        2 dir.   47.4       9.170  1.1       12.7    3.5
```

```
2                    422 dir.   47.0      -0.873  0.1      -1.2   -0.3
3                    424 dir.   30.3       7.588  1.1      14.8    7.2
```

3.6.1 Test on normal distribution of homogenized residuals

Repeated observations often display a normal frequency distribution. Residual of observed quantities are linear function real errors. From presumption of normal ditribution of real errors follows that homogenized residuals should have normal distribution as well.

Program `gama-local` estimates mean value $E(\tilde{v})$ and estimate of variance $V(\tilde{v})$ for the vector of homogenized residuals

$$E(\tilde{v}) = \frac{1}{N}\sum_{i=1}^{N}\tilde{v}_i, \qquad V(\tilde{v}) = E(\tilde{v}^2) - (E(\tilde{v}))^2.$$

Vector of homogenized residuals transforms to *normalized (standardized) vector of residuals*

$$\nu_i = \frac{\tilde{v}_i - E(\tilde{v})}{\sqrt{V(\tilde{v})}}.$$

Using Kolmogorov-Smirnov test program `gama-local` verifies assumption of normality of elements of vector ν. Result of the test is a value saying what is the probability that elements of vector ν are a random sample form normal distribution $N(0, 1)$.

Kolmogorov-Smirnov test for one sample is based on maximal difference between empirical and theoretical cumulative distribution function (normal distribution $N(0, 1)$ in our case). For random sample with N elements $X_1, X_2, \ldots, X_N$ from the population with cumulative distribution function $F(x)$ we form empirical cumulative distribution function

$$S_N(x) = (\text{number of elements } X_1, X_2, \ldots, X_N \text{ which are} \leq x)/N,$$

If we denote

$$D = \max_{-\infty<x<\infty} |S_N(x) - F(x)|,$$

the testing criterion $D\sqrt{N}$ has limit of Kolmogorov-Smirnov distribution. Some critical values of testing criterion $D\sqrt{N}$ computed from the KS distribution are given in the following table

	0.005	0.010	0.025	0.050	0.100
Lower	0.42	0.44	0.48	0.52	0.57
Upper	1.73	1.63	1.48	1.36	1.22

3.7 Identification of weak network elements

When planning observations in a geodetic network we always try to guarantee that all observed elements are checked by other measurements. Only with redundant measurements it is possible to adjust observations and possibly remove blunders that might otherwise totaly corrupt the whole set of measurements. Apart from sufficient number of redundant observations the degree of control of single observed elements is given by the network configuration, ie. its geometry.

Less controlled observations represent weak network elements and they can in extreme cases even disable detection of gross observational errors as it is in the case of uncontrolled observations. There are two limit cases of observation control

fully controlled observation as is for example an observed distance between two fixed points (standard deviation of the adjusted element is zero; standard deviation of the residual equals to the standard deviation if the observation) and

uncontrolled observations as is a free polar bar for example (standard deviation of adjusted value is equal to standard deviation of observed quantity; residual and standard deviation of the residual are zero).

Weakly controlled or uncontrolled observations can result even from elimination of certain suspisios observations during analysis of adjusment.

Standard deviation of adjusted observations is less than standard deviation of the measurement. Degree of observation control in network is defined as coefficient

$$f = 100\frac{m_\ell - m_L}{m_\ell}, \tag{8}$$

where m_ℓ is standard deviation of observed quantity and m_L is standard deviation computed from a posteriori reference standard deviation m_0 . We consider observed network element to be

uncontrolled if $f < 0.1$ (in listing marked with letter `u`),

weakly controlled if $0.1 \leq f < 5$ (in listing marked with letter `w`).

3.8 Estimation of real errors

Acording to previous section we can consider an observation to be controlled if its coefficient $f > 0.1$. Any controlled observation can be eliminated from the network without corrupting the network consistency—network reduced by one controlled observation can be adjusted and all unknown parameters can be compute without the eliminated observation.

Estimate of real error of i -th observation is defined as

$$\varepsilon_{\ell_i} = L_i^{red} - \ell_i, \tag{9}$$

where ℓ_i is value of i -th observation and L_i^{red} is value of i -th network element computed from adjusted coordinates and/or orientations of the reduced network. Similarly is defined the estimate of real error of a residual

$$\varepsilon_{v_i} = L_i^{red} - L_i. \tag{10}$$

Adjustment results are the best statistical estimate of unknown parameters that we have. This holds true even for adjustment of *reduced* network which is not influenced by real error of i -th observation. On favourable occasions differences (9) and (10) can help to detect blunders but to interpret these estimates as *real errors* is possible only with substantial exaggeration. These estimates fail when there are more than one significant observational error. Generally holds tha the weaker the element is controlled in netowrk the less reliable these estimates are.

Estimate of real error of an observation computes program `gama-local` as

$$\varepsilon_{\ell_i} = v_i/(p_i q_{v_i})$$

and estimate of real error of a residual as

$$\varepsilon_{v_i} = \varepsilon_{l_i} - v_i.$$

3.9 Test on linearization

Mathematical model of geodetic network adjustment in `gama-local` is defined as a set of known real-valued differentiable functions

$$\mathbf{L}^* = \varphi(\mathbf{X}^*), \tag{11}$$

where $\mathbf{L}^*$ is a vector of theoretical correct observations and $\mathbf{X}^*$ is a vector of correct values of parameters. For the given sample set of observations $\mathbf{L}$ and the unknown vector of residuals $\mathbf{v}$ we can express the estimate of parameters $\mathbf{X}$ as a nonlinear set of equations

$$\mathbf{L} + \mathbf{v} = \varphi(\mathbf{X}). \tag{12}$$

With approximate values $\mathbf{X_0}$ of unknown parameters

$$\mathbf{X} = \mathbf{X_0} + \mathbf{x}$$

we can linearize the equations (12)

$$\mathbf{L} + \mathbf{v} = \varphi(\mathbf{X_0}) + \left.\frac{\partial\varphi}{\partial\mathbf{X}}\right|_{\mathbf{X}=\mathbf{X_0}} \mathbf{x}$$

yielding the linear set of equations (1) where

$$\mathbf{A} = \left.\frac{\partial\varphi}{\partial\mathbf{X}}\right|_{\mathbf{X}=\mathbf{X_0}} \qquad \text{and} \qquad \mathbf{b} = \mathbf{L} - \varphi(\mathbf{X}_0).$$

Unknown parameters in `gama-local` mathematical model are points coordinates and orientation angles (transforming observed directions to bearings). The observables described by functions (12) belong into two classes

linear observables: horizontal and slope distances, height differences, control coordinates and vectors (coordinate differences),

angular observables: directions, horizontal and zenith angles.

Internally in `gama-local` unknown corrections to linear observables are computed in millimeters and corrections to angular observables in centigrade seconds. To reflect the internal units in used all partial derivatives of angular observables by coordinates are scaled by factor $2000/\pi = 10^{-3} \times (200 \times 10^4/\pi)$.

When computing coefficients of project equations (1) we expect that approximate coordinates of points are known with sufficient accuracy needed for linearization of generally nonlinear relations between observations and unknown paramters. Most often this is true but not always and generally we have to check how close our approximation is to adjusted parameters.

Generally we check linearization in adjustment by double calculation of residuals

$$\mathbf{v}^i = \mathbf{A}\mathbf{x} - \mathbf{b},$$
$$\mathbf{v}^{ii} = \bar{\ell}(\bar{\mathbf{x}}) - \ell,$$

where in our notation $\mathbf{x}$ is vector of corrections of approximate unknown parameters $\mathbf{x_0}$, $\mathbf{b}$ vector of reduced observations, ℓ vector of observations and $\bar{\ell}(\bar{\mathbf{x}})$ is vector of adjusted observations conputed from adjusted coordiantes $\bar{\mathbf{x}} = \mathbf{x_0} + \mathbf{x}$. Disagreement $\mathbf{v}^i \neq \mathbf{v}^{ii}$ signals discrepancies in linearization.

Program `gama-local` similarly computes and tests differences in values of adjusted observations once computed from residuals and once from adjusted coordinates. For measured directions and angles `gama-local` computes in addition transverse deviation corresponding to computed angle difference in the distance of target point (or the farther of two targets for angle). As a criterion of bad linearization is supposed positional deviation greater or equal to 0.0005 millimetres.

Example

```
Test of linearization error
***************************

Diffs in adj. obs from residuals and from adjusted coordinates
**************************************************************

 i standpoint     target          observed     r        difference
================================= hodnota = [mm|cc] = [cc] == [mm]=

 2 3022184030 3022724008 dist.   28.39200    -7.070          -0.003
 3            3022724002 dist.   72.30700   -18.815          -0.001
 7            3000001063 dir.  286.305200    11.272  -0.002  -0.001
 8            3022724008 dir.  357.800600   -23.947   0.037   0.002
```

From the practical point of view it might seem that the tolerance 0.0005 mm for detecting poor linearization is too strict. Its exceeding in program `gama-local` results in repeated adjustment with substitute adjusted coordinates for approximate. Given tolerance was chosen so strict to guarantee that listed output results would never be influenced by linearization and could serve for verification and testing of numerical solutions produced by other programs.

Implicitly coordinates of constrained points are not changed in iterative adjustments. This feature can be changed in XML input data by setting `<parameters update-constrained-coordinates = "yes" />` (see Section 2.5 [Network parameters], page 7).

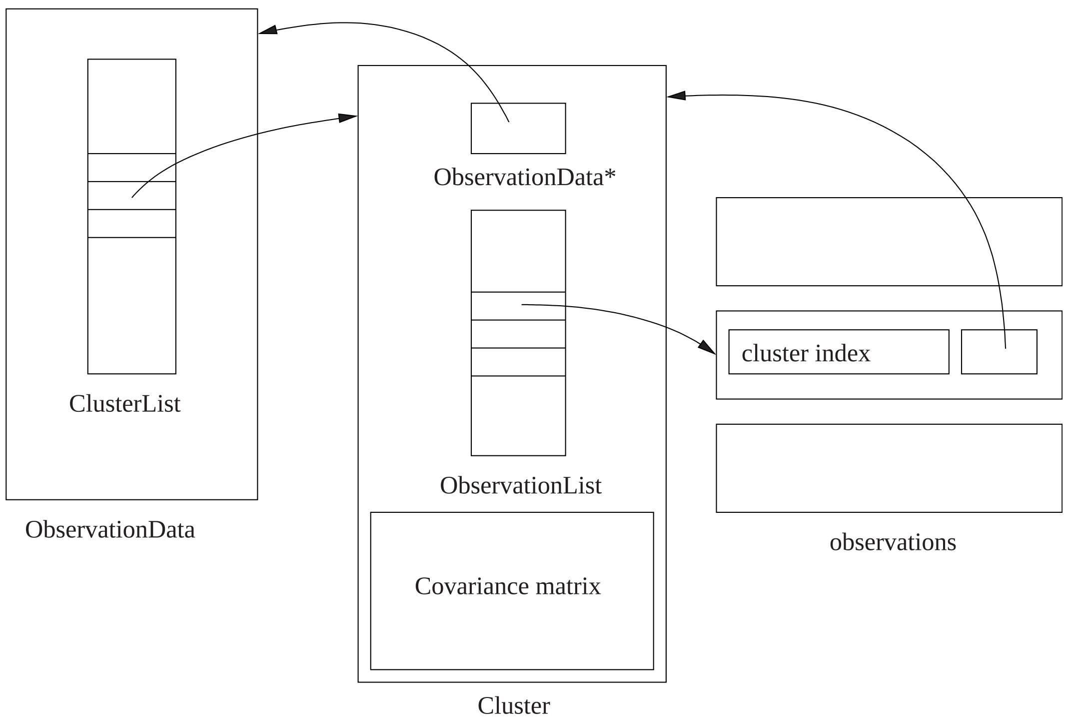
Iterated adjustement with successive improvement of approximate coordinates converges usually even for gross errors in initial estimates of unknown coordinates. If the influence of linearization is detected after adjustment, quite often only one iteration is sufficient for recovering.

For any automatically controlled iteration we have to set up certain stopping criterion independent on the convergence and results obtained. Program `gama-local` computes iterated adjustment three times at maximum. If the bad linearization is detected even after three readjustments it signals that given network configuration is somehow suspicious.

4 Data structures and algorithms

4.1 Observation data and points

The Gama observation data structures are designed to enable adjustment of any combination of possibly correlated observations. At its very early stage Gama was limited to adjustment of uncorrelated observations. Only directions and distances were available and observable's weight was stored together with the observed value in a single object. A single array of pointers to observation objects was sufficient for handling all observations. So called *orientation shifts* corresponding to directions measured form a point were stored together with coordinations in *point objects*.



To enable adjustment of possibly correlated observations (like angles derived from observed directions or already adjusted coordinates from a previous adjustment) Gama has come with the concept of *clusters*. Cluster is an object with a common variance-covariance matrix and a list of pointers to observation objects (distances, directions, angles, etc.). Weights were removed from observation objects and replaced with a pointer to the cluster to which the observation belong. All clusters are joined in a common object `ObservationData`; similarly to observations, each cluster contains a pointer to its parent `Observation Data` object. *Orientation shifts* were separated from coordinates and are stored in the cluster containing the bunch of directions and thus number of orientations is not limited to one for a point.

This organisation of observational information has proved to be effective. Template classes `ObservationData` and `Cluster` are used as base classes both in `gama-local` and `gama-w3`

```
template <typename Observation>
```

```
  class ObservationData
  {
  public:
    ClusterList<Observation>  CL;

    ObservationData();
    ObservationData(const ObservationData& cod);
    ~ObservationData();

    ObservationData& operator=(const ObservationData& cod);
    template <typename P> void for_each(const P& p) const;
  };

template <typename Observation>
  class Cluster
  {
  public:
    const ObservationData<Observation>*  observation_data;
    ObservationList<Observation>         observation_list;
    GaMaLib::Cov                         covariance_matrix;

    Cluster(const ObservationData<Observation>* od);
    virtual ~Cluster();

    virtual Cluster* clone(const ObservationData<Observation>*) const = 0;
    double stdDev(int i) const;
    int size() const;
    void update();
    int  activeCount() const;
    GaMaLib::Cov  activeCov() const;
  };
```

The following template class `PointBase` for handling point information is used in `gama-w3`. The template class `PointBase` relies internally on `std::map` container but comes with its own interface (in `gama-local` `std::map` was used directly for storing points).

```
template <typename Point>
  class PointBase
  {
    typedef std::map<typename Point::Name, Point*>  Points;

  public:
    PointBase();
    PointBase(const PointBase& cod);
    ~PointBase();

    PointBase& operator=(const PointBase& cod);
```

```
    void put(const Point&);
    void put(Point*);
    Point*       find(const typename Point::Name&);
    const Point* find(const typename Point::Name&) const;
    void erase(const typename Point::Name&);
    void erase();

    class const_iterator;
    const_iterator  begin();
    const_iterator  end  ();

    class iterator;
    iterator  begin();
    iterator  end  ();
  };
```

Template classes `ObservationData` and `PointBase` are defined in namespace `GNU_gama` and are located in the source directory `gnu_gama`.

4.2 Supported ellipsoids

id	a	b, 1/f, f	description	
airy	6377563.396	6356256.910	Airy ellipsoid 1830	[4]
airy_mod	6377340.189	6356034.446	Modified Airy	[4]
apl1965	6378137	298.25	Appl. Physics. 1965	[4]
andrae1876	6377104.43	300.0	Andrae 1876 (Denmark, Iceland)	[4]
australian	6378160	298.25	Australian National 1965	[3]
bessel	6377397.15508	6356078.96290	Bessel ellipsoid 1841	[1]
bessel_nam	6377483.865	299.1528128	Bessel 1841 (Namibia)	[4]
clarke1858a	6378361	6356685	Clarke ellipsoid 1858 1st	[3]
clarke1858b	6378558	6355810	Clarke ellipsoid 1858 2nd	[3]
clarke1866	6378206.4	6356583.8	Clarke ellipsoid 1866	[3]
clarke1880	6378316	6356582	Clarke ellipsoid 1880	[3]
clarke1880m	6378249.145	293.4663	Clarke ellipsoid 1880 (modified)	[4]
cpm1799	6375738.7	334.29	Comm. des Poids et Mesures 1799	[4]
delambre	6376428	311.5	Delambre 1810 (Belgium)	[4]
engelis	6378136.05	298.2566	Engelis 1985	[4]
everest1830	6377276.345	300.8017	Everest 1830	[4]
everest1848	6377304.063	300.8017	Everest 1948	[4]
everest1856	6377301.243	300.8017	Everest 1956	[4]
everest1869	6377295.664	300.8017	Everest 1969	[4]

everest_ss	6377298.556	300.8017	Everest (Sabah and Sarawak)	[4]
fisher1960	6378166	298.3	Fisher 1960 (Mercury Datum)	[3] [4]
fisher1960m	6378155	298.3	Modified Fisher 1960	[3] [4]
fischer1968	6378150	298.3	Fischer 1968	[4]
grs67	6378160	298.2471674270	GRS 67 (IUGG 1967)	[4]
grs80	6378137	298.257222101	Geodetic Reference System 1980	[1]
hayford	6378388	297	Hayford 1909 (International)	[1] [3]
helmert	6378200	298.3	Helmert ellipsoid 1906	[3]
hough	6378270	297	Hough	[4]
iau76	6378140	298.257	IAU 1976	[4]
international	6378388	297	International 1924 (Hayford 1909)	[1] [3]
kaula	6378163	298.24	Kaula 1961	[4]
krassovski	6378245	298.3	Krassovski ellipsoid 1940	[1]
lerch	6378139	298.257	Lerch 1979	[4]
mprts	6397300	191.0	Maupertius 1738	[4]
mercury	6378166	298.3	Mercury spheroid 1960	[3]
merit	6378137	298.257	MERIT 1983	[4]
new_intl	6378157.5	6356772.2	New International 1967	[4]
nwl1965	6378145	298.25	Naval Weapons Lab., 1965	[4]
plessis	6376523	6355863	Plessis 1817 (France)	[4]
se_asia	6378155	6356773.3205	Southeast Asia	[4]
sgs85	6378136	298.257	Soviet Geodetic System 85	[4]
schott	6378157	304.5	Schott 1900 spheroid	[3]
sa1969	6378160	298.25	South American Spheroid 1969	[3]
walbeck	6376896	6355834.8467	Walbeck	[4]
wgs60	6378165	298.3	WGS 60	[4]
wgs66	6378145	298.25	WGS 66	[4]
wgs72	6378135	298.26	WGS 72	[4]
wgs84	6378137	298.257223563	World Geodetic System 1984	[1]

[1] Milos Cimbalnik - Leos Mervart: Vyssi geodezie 1, 1997, Vydavatelstvi CVUT, Praha

[2] Milos Cimbalnik: Derived Geometrical Constants of the Geodetic Reference System 1980, Studia geoph. et geod. 35 (1991), pp. 133-144, NCSAV, Praha

[3] Glossary of the Mapping Sciences, Prepared by a Joint Committe of the American Society of Civil Engineers, American Congress on Surveying and Mapping and American Society for Photogrammetry and Remote Sensing (1994), USA, ISBN 1-57083-011-8, ISBN 0-7844-0050-4

[4] Gerald Evenden: proj - forward cartographic projection filter (rel. 4.3.3), http://www.remotesensing.org/proj

4.3 Transformation from spatial to geographical coordinates

Spatial coordinates (X, Y, Z) can be easily computed from geographical ellipsoidal coordinates (B, L, H), where B is geographical latitude, L geographical longitude and H is elliposidal height, as

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} (N+H)\cos B\cos L \\ (N+H)\cos B\sin L \\ (N(1-e^2)+H)\sin B \end{pmatrix}$$

where $N = a/\sqrt{1-e^2 sin^2 B}$ is the radius of curvature in the prime vertical, $e^2 = (a^2-b^2)/a^2$ is the first eccentricity for the given rotational ellipsoid (spheroid) with semi-major axis a and semi-minor axis b.

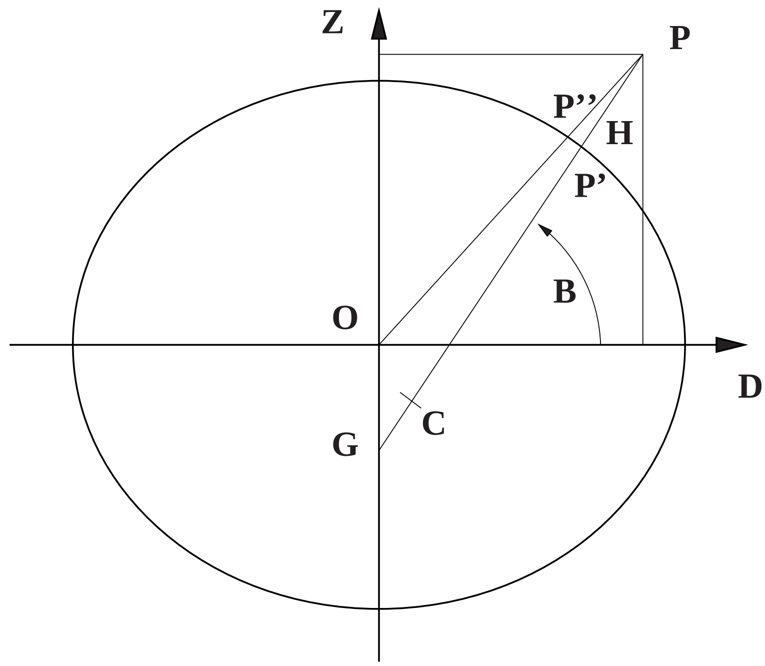
In the case of coordiante transformation from (X, Y, Z) to (B, L, H), the longitude is given by the formula

$$\tan L = Y/X.$$

Now we can introduce $D = \sqrt{X^2+Y^2}$, so that the cartesian system become (D, Z). Coordinates B and H are then usually computed by iteration with some starting value of B_0, for example $\tan B_0 = Z/D/(1-e^2)$,

$$\tan B_i = Z/D + \frac{N_{i-1}}{(N_{i-1}+H_{i-1})} e^2 \tan B_{i-1}, \quad H_i = D/\cos B_{i-1} = Z/\sin B_{i-1} - N(1-e^2)$$

B. R. Bowring described a closed formula[1] that is more effective and sufficiantly accurate and that is used in GNU Gama.



The centre of curvature C of the spheroid corresponding to P' is the point

$$(e^2 a \cos^2 u, -e'^2 b \sin^3 u),$$

where $e'^2 = (a^2 - b^2)/b^2$ is second eccentricity and u is the parametric latitude of the point P', $(1 - e^2)N \sin B = b \sin u$. Therefore

$$\tan B = \frac{Z + e'^2 b \sin^3 u}{D - e^2 a \cos^3 u}.$$

This is clearly an iterative solution; but it has been found that this formula is extremely accurate using the single first approximation for u for the $\tan u = (Z/D)(a/b)$. Maximum error in earth bound region is 3e-8 of sexadecimal arc seconds (5e-7 millimetres); maximum is 0.0018" (0.1 millimetres) at height H = 2a.

4.4 Class `g3::Model`

g3::model documentation shall come here ...

```
namespace GNU_gama {  namespace g3 {


  class Model {
```

[1] B. R. Bowring: Transformation from spatial to geographical coordinates, Survey Review XXIII, 181, July 1976

```
  public:

    typedef GNU_gama::PointBase<g3::Point>              PointBase;
    typedef GNU_gama::ObservationData<g3::Observation>  ObservationData;

    PointBase           *points;
    ObservationData     *obs;

    GNU_gama::Ellipsoid  ellipsoid;


    Model();
    ~Model();

    Point* get_point(const Point::Name&);
    void   write_xml(std::ostream& out) const;
    void   pre_linearization();
}}
```

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